# Interbank Market Freezes and Creditor Runs*

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**Abstract**

We model the interplay between trade in the interbank market and creditor runs on financial institutions. We show that the feedback between them can amplify a small shock into "interbank market freezing" with "liquidity evaporating". Credit crunches of the interbank market drive up the interbank rate. For an individual institution, a higher interbank rate — meaning a higher funding cost — results in more severe coordination problems among creditors in debt rollover decisions. Creditors thus behave more conservatively and run more often. Facing an increased chance of creditor runs, institutions demand more and supply less liquidity, tightening the interbank market. Our model demonstrates that banking crises arise from a shrinking of the pool of aggregate liquidity.

*JEL classification*: G01; G21; D83; D53

*Keywords*: Interbank market, creditor runs, insolvency risk, illiquidity risk, global games

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# 1 Introduction

A salient feature of the recent financial crisis of 2007-2009 was s*ystemic* creditor runs on financial institutions. Short-term creditors of the institutions rushed for the exit and liquidity evaporated abruptly. The modern-day bank runs that occurred in the shadow banks illustrated vividly how liquidity could suddenly dry up.[1] Covitz et al. (2013) document that the runs on asset-backed commercial paper (ABCP) programs led to outstanding ABCP falling by $400 billion (one-third of the existing amount) during the second half of 2007. Duygan-Bump et al. (2013) document that the runs on prime money market funds (MMFs) caused asset value to shrink from $1300 billion to $900 billion within one week after the collapse of Lehman Brothers. Notably, systemic creditor runs coincided with the "freezing" of the interbank market and were strongly correlated with the macroeconomic fundamental shocks — the ABX index (Gorton and Metrick (2011) and Covitz et al. (2013)). The LIBOR-OIS spread (a primary measure of interbank lending rates) increased to over 300 bps at the peak of the crisis, in contrast to the pre-crisis level of less than 10 bps.[2]

This paper develops a model to demonstrate the interdependence of trade in the interbank market and creditor runs on financial institutions. Our model explains the joint occurence of interbank market freezes and systemic creditor runs.

The credit risk of debt in a financial institution can be decomposed into two parts: fundamental (insolvency) risk and coordination (illiquidity) risk. In our framework, the illiquidity risk is endogenous, originating in the insolvency risk. When the fundamental (insolvency) risk increases, the coordination problem among short-term creditors becomes more severe and thus they are more likely to run on the institution, so the illiquidity risk also increases. In our model, the role of the interbank market is to allow banks in the financial system with idiosyncratic fundamental shocks to trade short-term funds to solve their illiquidity problem and thus mitigate potential creditor runs.

We develop a three-date model. At the initial date, each bank in the system makes its portfolio allocation: cash holdings and investment in a long-term illiquid asset (with a higher expected return). At the intermediate date, banks realize their idiosyncratic fundamental shocks (asset quality). A higher asset quality of a bank means that the bank's long-term asset will realize the high

[1] Runs on banks such as Northern Rock, Bear Stearns, Lehman Brothers, and others have been well recognized.

[2] Data are from the British Bankers' Association and Bloomberg.

cash flow with a greater probability at the final date. Creditors of a bank receive imperfect information about the asset quality of their bank at the intermediate date, based on which make their rollover decisions. Creditors face a coordination problem among themselves in rollover decisions.

We first solve the equilibrium at the intermediate date, that is, the equilibrium of a creditor-run game. In the benchmark case without an interbank market, we show that the probability of a creditor run for a bank is a decreasing function of the bank's *asset quality* and *cash holdings*. Intuitively, a higher asset quality means a lower insolvency risk of the bank at the final date, and thus creditors are more willing to roll over their lending. A higher cash holding means that the bank is more liquid and can withstand a larger proportion of creditors' interim withdrawals, so the coordination risk (illiquidity risk) of the debt is lower. Therefore, cash holdings and asset quality are *substitutes* in preventing a creditor run. In other words, a stronger bank, with fewer creditors calling loans in equilibrium, needs less cash to prevent a creditor run. This gives rise to the role of an interbank market, where banks facing heterogeneous fundamental shocks borrow and lend short-term funds among themselves. Earlier works such as Bhattacharya and Gale (1987) and Allen and Gale (2000) model the interbank market instead based on the assumption that banks face exogenous idiosyncratic liquidity shocks.

In the case with an interbank market, we show that the probability of a creditor run for an individual bank depends on the bank's asset quality as well as the funding condition (i.e., borrowing rate) in the interbank market. The funding condition affects how much liquidity the bank can raise from the interbank market and thus how much total liquidity the bank can access. The funding condition of the interbank market, in turn, depends on other banks' status and their willingness to lend. Therefore, the equilibrium in the system, which we call the creditor run-interbank market equilibrium, has two interacting components: the creditor-run equilibrium *within* a bank and the interbank-market equilibrium *across* banks. The interbank-market equilibrium determines the interbank rate, which affects creditors' rollover decision in an individual bank. Conversely, creditors' rollover decision affects the status of each bank and hence the equilibrium of the interbank market.

Then, we solve the equilibrium at the initial date; in doing so the cash holdings of banks and the notional (promised) interest rate to creditors are endogenized. We consider both the constrained second-best equilibrium and the competitive equilibrium.

Finally, we study our model under an aggregate shock, which has implications for the occurrence of a crisis and amplification. A crisis occurs when an adverse *aggregate shock* to asset quality hits the economy. Concretely, when some banks (perhaps high-quality banks) realize worse asset quality than they expect, they need more liquidity to mitigate their illiquidity problems. This decreases the supply of short-term funds in the interbank market, driving up the interbank rate. As the interbank rate increases, *ceteris paribus*, an individual bank can raise less liquidity in the interbank market and thus becomes less capable of meeting its creditors' early withdrawals. This triggers higher *coordination risk* among creditors in a rollover, who then run more often. All banks face such a problem (to different extents) because of information asymmetry of creditors about their bank's fundamentals (asset quality). In anticipation of this, banks demand more and supply less liquidity in the interbank market to protect themselves, leading to an even higher interbank rate. In short, an adverse aggregate shock triggers a reinforcing spiral of a rise in the interbank rate, a more severe coordination problem among creditors in debt rollover decisions, and a greater demand for liquidity of banks. Moreover, the adverse aggregate shock can trigger multiple equilibria under self-fulfilling expectations. The amplifications result in systemic creditor runs occurring simultaneously with interbank market freezes.

Our model demonstrates that banking crises arise from a shrinking of the pool of aggregate liquidity, which helps explain some important phenomena in the recent crisis. For example, the recent crisis originated in the US, where there had been an asset fundamental shock — the subprime mortgage crisis; however, the first bank that suffered bank runs was Northern Rock, a UK bank. The reason, we emphasize, is that Northern Rock and the US institutions tapped the same short-term funding market (see the evidence in Shin (2009)).

As for policy implications, our model considers two ex post intervention measures: liquidity injections and public disclosure. First, when a negative aggregate shock hits, injections of liquidity into the financial system are crucial to break the vicious cycle of feedback. The debate on whether central banks should provide emergency liquidity assistance went on well before the recent crisis. Goodfriend and King (1988) (see also Bordo (1990), Kaufman (1991) and Schwartz (1992)) remark that banking policy was necessary at a time when financial markets were underdeveloped; however, "with sophisticated interbank markets, banking policy has become redundant". In other words, they argue that with a well-functioning interbank market, a solvent institution cannot be illiquid.[3] Our work shows that under market frictions (imperfect information) there exists a vicious cycle of feedback that can amplify a small shock in insolvency into a systemic crisis, which justifies ex post intervention. Our model stresses that when the banking system is hit by an aggregate shock, the purpose of government intervention with liquidity injections is not to save a single bank, but to influence the interaction (among banks and with creditors) within the system and thereby improve overall efficiency. Second, public disclosure can improve efficiency. If the government is informed of individual banks' asset quality (through, for example, stress tests), there exists an optimal degree of transparency in public disclosure. Neither too much nor too little disclosure is efficient.

**Related literature** Our paper is related to the research that uses global game methods to address illiquidity risk (e.g., Morris and Shin (2004a, 2009), Rochet and Vives (2004), Goldstein and Pauzner (2005), Liu and Mello (2011), Eisenbach (2011)).[4] Rochet and Vives (2004) and Goldstein and Pauzner (2005) are the first to apply global games to study bank runs. Morris and Shin (2009) build an analytical framework to decompose the creditor risk in a financial institution into insolvency risk and illiquidity risk. Bebchuk and Goldstein (2011) and Vives (2014) build general frameworks to incorporate the results in this literature and analyze policy measures. Our paper contributes to this literature in that we study the interplay between illiquidity risk and insolvency risk in a *financial system* context by considering the interbank market and analyze systemic effects that play a large role in a crisis. We explicitly model the interbank market, explain feedback loops between creditor runs and interbank trading, and examine the transmission of shocks across institutions through the interbank market.[5]

Closely related to our work are Rochet and Vives (2004) and Goldstein (2005). Rochet and Vives (2004) study a creditor-run model with global games for a single bank in the presence of an

[3] See Rochet and Vives (2004) and Freixas and Rochet (2008) for a comprehensive discussion on this debate.

[4] In a related model not using global games, He and Xiong (2012) study the intertemporal coordination problem among creditors.

[5] Liu and Mello (2011) study the optimal cash holdings in the presence of coordination risk for a single institution in a partial equilibrium framework. Eisenbach (2011) constructs a model to endogenize the liquidation value (or the "fire sales penalty" in the language of Vives (2014)) by assuming an exogenous demand curve of the asset. Our work differs from his in at least two important ways: first, we study and endogenize the *competitive* interbank market; second, the ex ante problem of banks is studied, so the aggregate liquidity in the system is *endogenous*.

interbank market, where the interbank market and the interbank rate (or the “fire sales penalty”) are exogenous. Among other results, their model shows that the creditor-run equilibrium is affected by the interbank rate. To a large extent, our model provides a general-equilibrium treatment of Rochet and Vives (2004) and endogenizes the interbank market in their model. We show that the creditor-run equilibrium in the system affects, and is in turn affected by, the interbank rate.

Goldstein (2005) models the twin crises phenomenon in a global-games framework, which integrates the banking sector in Goldstein and Pauzner (2005) with the currency market in Morris and Shin (1998). His model shows that strategic complementarities exist not only within a group of creditors or within a group of currency speculators, but also between the two groups. The additional type of complementarities generates a vicious circle between banking crises and currency crises. Close in spirit to his insight, our model shows that there exists a vicious circle between creditor runs and interbank market freezes. Our paper adds to Goldstein (2005) in the following way. Goldstein (2005) abstracts the bank sector to one single bank and models the currency market in a reduced form. We explicitly model the banking sector with a continuum of banks and endogenize the *competitive* interbank market. The interbank market in our model is a Walrasian economy amd not a game; the price — the interbank rate — plays a key role in determining the equilibrium. To this end, we emphasize the feedback loop between asset prices and creditor runs.

Our paper is related to Diamond and Rajan (2005) in that both papers demonstrate that banking crises can arise from a shrinking of the pool of available aggregate liquidity. There are no interbank market or coordination issues (panics) among creditors of a bank in Diamond and Rajan’s (2005) model. Different from their study and contributions, modeling the two-way feedback between trade in the interbank market and creditor runs is a key emphasis of our model. The feedback loop in our paper is between asset prices and *coordination risk*, i.e., a higher interbank rate (a lower asset price) means that a bank can withstand withdrawals by fewer creditors and is more fragile, thereby increasing the difficulty of coordination among creditors to not run. This is related to, but different from, the feedback between asset prices and the leverage or margin constraints in the models of Gromb and Vayanos (2002) and Brunnermeier and Pedersen (2009). In another related study, Iori, Jafarey and Padilla (2006) evaluate based on simulations how the statistical characteristics of a market’s constituents and their interconnectedness affect systemic risk of the interbank market, but with exogenous strategies of the participants and risk of liquidity shortages.

On the empirical side, Copeland et al. (2011), Gorton and Metrick (2011), and Krishnamurthy et al. (2012) provide evidence on repo runs. Gorton and Metrick (2011) argue that the "run on repo" played a key role in the collapse of the US shadow banking system. Krishnamurthy et al. (2012), on the other hand, argue that the contraction in repo funding flows from non-bank lenders to shadow banks was small; the magnitude was relatively insignificant compared with the contraction in ABCP. Our model implies the importance of distinguishing between two types of repo lending, non-bank to dealer repo lending (largely tri-party) and interbank repo lending between dealers (largely bilateral), and demonstrates the feedback between interbank market freezes (such as bilateral repo runs) and creditor runs (such as runs through ABCP and tri-party repo).

The rest of the paper is organized as follows. Section 2 sets up the model. Sections 3 and 4 present the equilibria. Section 5 studies the model under an aggregate shock. Section 6 discusses policy implications of the model. Section 7 concludes.

# 2 The model

The model has three dates: $T_0$, $T_1$, and $T_2$, and there is no time discount for simplicity. All agents are risk-neutral. We discuss banks, the interbank market and bank runs, in that order.

## 2.1 Banks

There is a continuum of banks (commercial or investment banks, or more broadly, institutions in the shadow banking system) with unit mass, indexed by $i \in [0, 1]$. Ex ante, at $T_0$, these banks are identical. Each bank has one unit of cash, which is financed by the owner of the bank (hereafter, equityholder or bankowner) as well as a continuum of short-term creditors (depositors) of total measure $F$, each contributing 1 unit of cash.[6] That is, the liability side of the balance sheet of a bank at $T_0$ includes debt of total face value $F$ and the equity value $1 - F$, where $0 < F < 1$. We can think of each bank as a regional or sectoral bank with its own investor (creditor) pool. A creditor's reserve value (opportunity cost) of lending is $R_0$ per unit of cash, where $R_0 \geq 1$.

[6] The short-term debt in our model may play the role of disciplining (Calomiris and Kahn (1991), Diamond and Rajan (2001)). For example, without the threat of a potential short-term debtholder run, the owner of a bank could take (off-equilibrium) actions that make the bank's asset more risky with a negative NPV. Kacperczyk and Schnabl (2010) provide indirect evidence for the disciplining role of short-term debt.

At $T_0$, a bank needs to make its investment portfolio allocation: cash holdings and investment in a long-term asset (technology). Each bank has access to a long-term risky asset (technology) with stochastic payoffs at $T_2$. The unit cost of a long-term asset at $T_0$ is 1. If a long-term asset is *physically* liquidated at $T_1$, it realizes $\varepsilon$ (close to 0) liquidation value. Denote the investment decision of a bank at $T_0$ by $(c, 1-c)$, where $c$ is the amount of cash holdings and $1-c$ is the amount of investment in the long-term asset. A bank's balance sheet at $T_0$ is represented by Figure 1, where $A^S$ and $A^L$ are the short-term liquid asset and long-term illiquid asset, respectively.

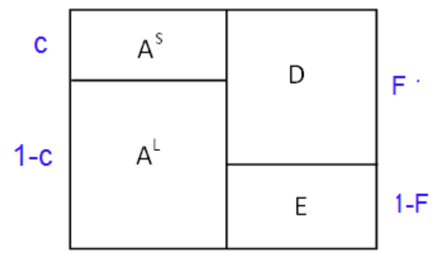


**Figure 1**: Balance sheet of banks at $T_0$

The uncertainty about the payoff of a bank's long-term asset will be resolved gradually, as shown in Figure 2. The uncertainty at $T_0$ is characterized by random variable $\theta$, interpreted as *asset quality*. $\theta$ has smooth density $g(\theta)$ and cumulative distribution function $G(\theta)$ in the support of clopen set $[-\infty, \infty]$. The mean of $\theta$ is $\mu$ and its standard deviation is $\sigma$. At $T_1$, the uncertainty about each individual bank's asset quality is resolved. Specifically, bank $i$ realizes its asset quality $\theta^i$, where $\theta^i$ is independently drawn from the identical probability distribution $\theta \sim g(\theta)$. That is, banks are identical ex ante but heterogeneous (with idiosyncratic shocks) at $T_1$. At $T_2$, one of the two cash flows, $\{0, X\}$, will be realized per unit of a long-term asset. The probability of realizing $X$ for an asset of quality $\theta^i$ is $\pi(\theta^i)$, where $\pi(\cdot)$ is a continuous and increasing one-to-one function, $\pi : [-\infty, \infty] \to [\underline{\pi}, \overline{\pi}]$, with $0 \leq \underline{\pi} < \overline{\pi} \leq 1$. We also assume that $\int_{-\infty}^{\infty} [X \cdot \pi(\theta)]\, g(\theta) d\theta > R_0$, which means that investing in a long-term asset is profitable ex ante at $T_0$.

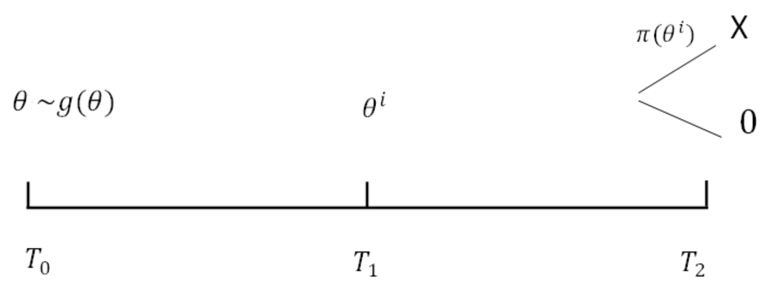


**Figure 2**: The uncertainty about the payoff of a long-term asset

In addition, we assume that asset payoff realizations at $T_2$ across banks are positively correlated (as in Holmstrom and Tirole (1997)).[7] Specifically, if a higher-$\theta$ bank's asset cannot deliver cash flow $X$ at $T_2$, then neither can a lower-$\theta$ bank's asset. The only purpose of this assumption is to ensure that the risk of debt of a higher-$\theta$ bank cannot be reduced (diversified away) when it acquires some assets from lower-$\theta$ banks. Concretely, we can make the following simple specification. Let the economy have a continuum of states at $T_2$, denoted by $\omega \in [0,1]$. The realization of the state will be drawn by the nature at $T_2$ uniformly, i.e., $\omega \sim U[0,1]$. An asset of quality $\theta$ at $T_1$ has state-dependent payoff at $T_2$ of $\widetilde{x}(\theta,\omega) = \begin{cases} X & \text{if } \omega \in [0,\pi(\theta)] \\ 0 & \text{if } \omega \notin [0,\pi(\theta)] \end{cases}$. Thus, we have i) $\Pr(\widetilde{x} = X|\theta) = \pi(\theta)$ and ii) $\widetilde{x}(\theta',\omega) \geq \widetilde{x}(\theta'',\omega)$ for $\theta' > \theta''$ under any state $\omega$, with strict inequality holding for some $\omega$.

The owner of a bank gets informed, at $T_1$, of the quality of his bank's long-term asset as well as the quality of assets of other banks. Empirical evidence suggests that bankers, as insiders, have information about each other's status (see, e.g., the findings in Afonso, Kovner and Schoar (2011)). But a creditor of a bank receives only a signal at $T_1$ regarding the quality of long-term asset of his own bank. Specifically, the signal received by creditor $j$ in bank $i$ (about asset quality $\theta^i$) at $T_1$ is $x^{ij} = \theta^i + \delta\epsilon^j$, where $\delta > 0$ is constant, and the individual-specific noise $\epsilon^j$ is distributed according to the smooth symmetric density $h(\cdot)$ with mean 0 (writing its c.d.f. as $H(\cdot)$). $\epsilon^j$ is i.i.d. across creditors of a bank, and each is independent of $\theta^i$. Like creditors, outsiders including a court cannot observe or verify the asset quality of a bank at $T_1$, as in the incomplete contract literature. Such a setup — the realization of idiosyncratic shocks being unverifiable — is employed in a large literature including Diamond and Dybvig (1983) and Bhattacharya and Gale (1987).

Creditors of a bank are offered demand-deposit-like contracts: if a creditor calls his loan at $T_1$, his claim is the par (face) value 1; if, instead, he decides to roll over his loan until $T_2$, the *notional (promised)* value of his claim is $R$, where $R$ is the gross interest rate, to be endogenized.[8]

For tractability, we use the following distributions and function specifications throughout the paper. The (prior) distribution of $\theta$ is assumed to be Gaussian. Specifically, $\theta \sim N(\mu, \sigma^2)$; that is,

[7] Holmstrom and Tirole (1997) argue that correlation may come from intermediaries' incentive of choosing correlated projects, or their specialized expertise in monitoring projects, or macroeconomic shocks.

[8] Without loss of generality, we normalize the interim notional claim at $T_1$ to 1. What matters for the model is the interest rate between $T_1$ and $T_2$, i.e., $R$.

$g(\theta) = \frac{1}{\sigma}\phi(\frac{\theta-\mu}{\sigma})$, where $\phi(\cdot)$ stands for the p.d.f. of the standard normal distribution. The c.d.f of $\theta$ is thus $G(\theta) = \Phi(\frac{\theta-\mu}{\sigma})$, where $\Phi(\cdot)$ is the c.d.f. of the standard normal distribution. Furthermore, we assume that $\pi(\theta) = \underline{\pi} + (\overline{\pi} - \underline{\pi})\Phi(\frac{\theta-\mu_\pi}{\sigma_\pi})$, where $\mu_\pi$ and $\sigma_\pi$ are parameters. If $\mu_\pi = \mu$ and $\sigma_\pi = \sigma$, it is easy to check that $\pi(\cdot)$, as a function of $\theta$, is *uniformly* distributed with $\pi(\theta) \sim U[\underline{\pi}, \overline{\pi}]$; that is, the probability that a bank's long-term asset will realize $X$ at $T_2$ as perceived at $T_0$ is uniformly distributed within $[\underline{\pi}, \overline{\pi}]$. The distribution of signal noise $\epsilon^j$ is assumed to be the standard normal: $\epsilon^j \sim N(0,1)$; that is, $h(\cdot)$ is $\phi(\cdot)$ and $H(\cdot)$ is $\Phi(\cdot)$.

## 2.2 Interbank market

An interbank credit market opens at $T_1$, where banks can borrow and lend short-term funding among themselves. For instance, the interbank market can be the (bilateral) repo market. Denote the total (maximum) amount of cash available to a bank at $T_1$ by $c_1^T$, which includes the bank's own cash holdings ($c$) and the net borrowing from the interbank market. It will become clear later that $c_1^T$ is a function of asset quality $\theta^i$.

Two remarks are in order. First, in our general equilibrium model, when a bank raises liquidity at $T_1$ the liquidity must come from other banks *in the system*. Interbank borrowing means that a bank obtains some cash at $T_1$ from other banks but gives up some of its cash flow (from its long-term projects) at $T_2$ to repay those banks. That is, there is an exchange between $T_1$-cash flow and $T_2$-cash flow. The interbank rate is defined as the ratio of the expected $T_2$-cash flow (repayment) to the $T_1$-cash flow (borrowing). Second, in the spirit of one interpretation in Rochet and Vives (2004), we assume that the interbank market is informationally efficient, in the sense that lender banks have information about the asset quality of the borrower bank. Hence, the amount of liquidity that a bank can raise in the interbank market depends on the bank's asset quality. This assumption has strong empirical support (see the findings of Afonso, Kovner and Schoar (2011)).[9] The interbank market often operates as the bilateral repo market, so there can be some search or process by which a bank finds the asset quality of the counterparty. The existence of LIBOR (a

[9] In examining the recent financial crisis, Afonso, Kovner and Schoar (2011) do not find evidence that increased information asymmetry (averse selection) leads to higher risk premiums in the market overall. Rather, they find that loan terms become more sensitive to borrower characteristics and there is greater divergence in the cost of borrowing and in access to liquidity between weaker and stronger banks.

proxy for the competitive rate) also makes the interbank market resemble a central exchange-based market in some respects (see the discussions in Duffie and Stein (2015)).

We acknowledge that the market of interbank lending is complex. It has mixed features of a central exchange-based market (e.g., the existence of LIBOR) and an over-the-counter (OTC) market. For tractability and to focus on the main message of the paper, we abstract away from the complexity by studying a competitive interbank market and assuming no information asymmetry between banks. Explicitly modeling the search process in the interbank market and the existence of some degree of information asymmetry between banks is left to future research.

## 2.3 Bank runs

If a bank has greater than $c_1^T$ creditors declining to roll over their lending at $T_1$, the bank cannot fulfill its debt obligation and will consequently fail — a creditor run.[10]

Following the work of Rochet and Vives (2004) (as well as Morris and Shin (2009)), we use a simplified payoff structure of the creditor-run game. Specifically, as in their work, we assume that each creditor of a bank is an institutional investor (a fund), run by its fund manager. A fund manager has the following compensation scheme. If the fund manager calls the fund's investment at $T_1$, his payoff is a constant $w_0$, or the face value 1 multiplied by proportion $w_0$. This could be because, as will be shown later, calling loans at $T_1$ makes a creditor either fully recover the face value of investment 1 or suffer a small loss. $w_0$ can also be interpreted as the fund manager's monetary compensation after deducting the non-pecuniary penalties (e.g., reputation loss). If, instead, the fund manager holds the investment, his payoff is the fund's return multiplied by $\gamma$. Throughout the paper, we can assume that $w_0 \to 0$ and $\gamma \to 0$, meaning that the compensation to a creditor's fund manager is negligible relative to the total payoff to the creditor. The ratio $\frac{w_0}{\gamma}$ is called the "outside option ratio" in Morris and Shin (2009). Table 1 shows the simplified payoff structure (for a fund manager) of the creditor-run game, where the bank has asset quality $\theta$ at $T_1$.

[10] The physical liquidation value of illiquid assets is $\varepsilon \to 0$. The bank fails as soon as physical liquidation starts.

| | Total calling no greater than $c_1^T$ (bank survives) | Total calling greater than $c_1^T$ (bank fails ) |
|---|---|---|
| Hold | $\begin{cases} \gamma R & (\text{prob } \pi(\theta)) \\ 0 & (\text{prob } 1-\pi(\theta)) \end{cases}$ [11] | 0 |
| Call | $w_0$ | $w_0$ |

**Table 1**: Creditor-run payoff structure

By introducing a third party, fund managers, we have a discrete-state payoff structure of the creditor-run game. This simplified structure captures the following key feature of the creditor-run game: if the proportion of creditors of a bank calling the loans is higher than $\frac{c_1^T}{F}$, the optimal strategy for an individual creditor is to also 'call'; if, however, the proportion of creditors calling the loans is less than $\frac{c_1^T}{F}$, the optimal strategy for an individual creditor is likely to also 'hold'. The simplification in the payoff structure is a convenient way to deal with the fact that the property of global strategic complementarities fails to exist in a creditor-run game; see Goldstein and Pauzner (2005) for the full continuous-state payoff structure without simplification.[12]

If we substitute $w_0 = 1$ and $\gamma = 1$ into Table 1, the payoff structure then describes the payoff to a bank creditor without assuming the involvement of a fund manager. In this case, the simplified payoff structure is a *close approximation* of the full continuous-states payoff structure in Goldstein and Pauzner (2005) (see Eq. (A.1) below).

## 2.4 Timeline

Without loss of generality, we assume that at $T_2$ the principal and interest payment to (existing) bank creditors, who are offered demand-deposit-like contracts, is senior to the repayment to

[11] We will verify that if and only if the long-term asset of a bank pays $X$ at $T_2$, the interest $R$ is realized. In other words, a fund manager obtains $\gamma R$ with probability $\pi(\theta)$ conditional on the bank's surviving to $T_2$.

[12] See also Dasgupta (2004) and Liu and Mello (2012).

counterparty banks for the interbank borrowing.[13] Figure 3 summarizes the timeline.[14]

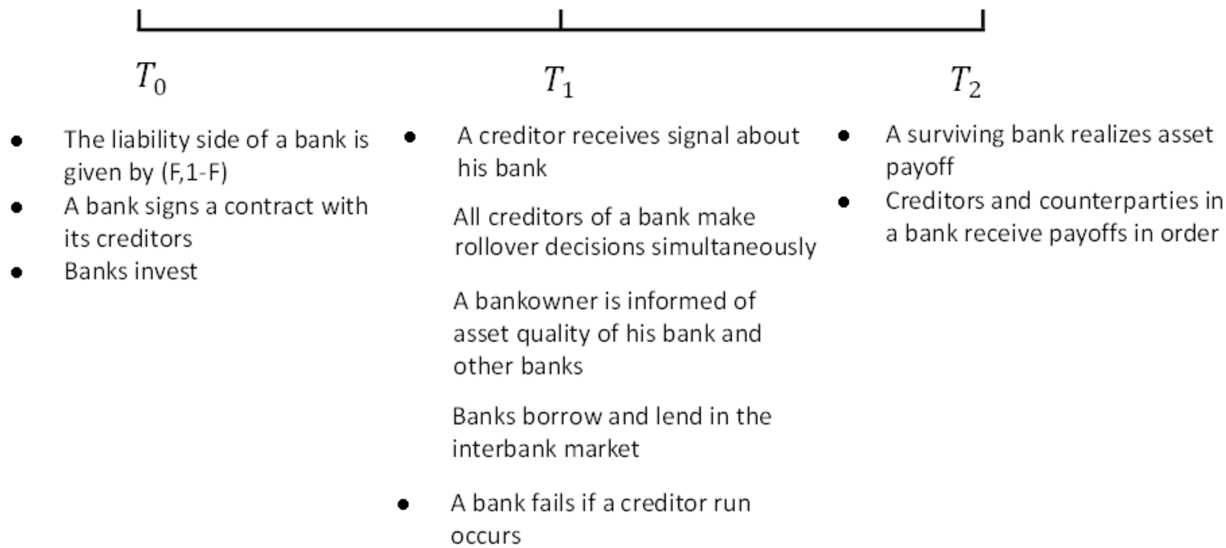


**Figure 3**: Timeline

# 3 Equilibrium at $T_1$ — creditor run-interbank market equilibrium

We conduct analysis by backward induction, from $T_1$ to $T_0$. In this section, for a given $c$ and $R$, which are set at $T_0$, we work out the equilibrium at $T_1$.

At $T_1$, creditors of a bank need to make their rollover decisions; that is, they play a creditor-run game. We are interested in the equilibrium where every creditor uses a threshold (monotone) strategy. The strategy is given by

$$x^{ij} \longmapsto \begin{cases} Call & x^{ij} < x^* \\ Hold & x^{ij} \geq x^* \end{cases},$$

[13] This assumption is purely for tractability (see Appendix D.1 for the robustness check). It is to make the cash flow pattern for each claimant simple, so that we can focus on the core of our mechanism. In fact, if interbank lenders are senior to creditors, the bank can default to a creditor at $T_2$ (even if the bank is liquid and thus does not default at $T_1$), in which case the cash flow pattern for a creditor would be very complicated and it would be difficult to make his participation condition (IR) tractable although the model result does not change qualitatively.

[14] At $T_1$, the rollover decision of creditors can be made either before or simultaneously with the interbank market decision of banks. Creditors do not know the interbank rate when they make their rollover decisions, and thus they must form expectations about it.

where $x^{ij}$ is the information of creditor $j$ in bank $i$ and $x^*$ is the rollover threshold.[15] Since banks are identical to creditors (putting aside their private signals), we consider the *symmetric* equilibrium in which creditors of all banks use a common strategy, i.e., the threshold $x^*$ is not bank-specific. As the prior distribution, $\theta \sim N(\mu, \sigma^2)$, is equivalent to public information about the asset quality for a creditor, a creditor's strategy $x^*$ depends on public information (mean $\mu$). We focus on the case of $0 << \frac{\delta}{\sigma} << +\infty$; that is, both private information and public information are valuable.

We also make one technical assumption here.

**Assumption 1** (**Upper dominance region**) There exists an *upper dominance region* $x^{ij} \in [x^{*U}, \infty)$ in which holding is the dominant strategy for a creditor. Specifically, we assume that when a bank's realized asset quality $\theta$ at $T_1$ is above a sufficiently high threshold, $\theta^U$, the government will be informed of its asset quality and is willing to bail it out (e.g., provide liquidity support) in the event that the bank cannot satisfy its creditors' early withdrawals on its own and seeks the government's support. That is, a bank never fails at $T_1$ when its $\theta \in [\theta^U, \infty]$. Therefore, if a creditor's signal is in the upper region $x^{ij} \in [x^{*U}, \infty)$, he does not run, no matter what his belief regarding the behavior of other creditors is, where $x^{*U}$ is a function of $\theta^U$ to be specified.

The assumption of the existence of an upper dominance region for the bank-run game follows Goldstein and Pauzner (2005). Note that like the effect of deposit insurance in Diamond and Dybvig (1983), the bailout will never *actually* be needed in equilibrium. That is, in equilibrium, when $\theta \geq \theta^U$ the bank can always satisfy its creditors' early withdrawals on its own, which will become clear later.

We first consider the benchmark equilibrium in the absence of an interbank lending market, and then consider the equilibrium in the presence of one.

## 3.1 Benchmark equilibrium at $T_1$ without an interbank lending market

In this case, we assume that banks are in autarky and there is no interbank lending market at $T_1$. So, $c_1^T = c$. For this autarky case, we can treat the system as having a *representative* bank.

[15] In the finance literature on applications of global games, the threshold (monotone) equilibrium is of primary interest. For example, Morris and Shin (2004b, 2009) consider only threshold (monotone) equilibria. The restriction to threshold strategies can be without loss of generality (see Vives (2014)).

We solve the creditor-run game equilibrium at $T_1$ of the representative bank for a given $c$ and $R$. We define another threshold $\widehat{\theta}$, which is the equilibrium bank failure threshold; that is, if and only if the bank's realized fundamental value $\theta$ is below $\widehat{\theta}$ does the bank fail at $T_1$.

For a given $\widehat{\theta}$, we consider the position of a creditor whose decision proxy is his fund manager. The *marginal* creditor (fund manager) in the bank who receives signal $x^*$ is indifferent to holding or calling, so we have

$$\int_{-\infty}^{\widehat{\theta}} 0 \cdot h_g(\theta|x^*)d\theta + \int_{\widehat{\theta}}^{\infty} (\gamma R) \cdot \pi(\theta) \cdot h_g(\theta|x^*)d\theta = w_0, \tag{1}$$

where $h_g(\theta|x^*)$ is the posterior (conditional) density with the prior being $g(\cdot)$. The right-hand side (RHS) of (1) is the payoff of calling. The left-hand side (LHS) of (1) expresses the payoff for the marginal creditor (fund manager) when he decides to roll over: if $\theta < \widehat{\theta}$, the bank cannot survive to $T_2$ and he gets nothing — the first term; conditional on the bank's surviving to $T_2$, his expected payoff is $(\gamma R) \cdot \pi(\theta)$ for a given realization of $\theta$ — the second term.[16] Equation (1) can be rewritten as

$$\int_{-\infty}^{\widehat{\theta}} 0 \cdot h_g(\theta|x^*)d\theta + \int_{\widehat{\theta}}^{\infty} (R \cdot \pi(\theta)) \cdot h_g(\theta|x^*)d\theta = \frac{w_0}{\gamma}. \tag{1'}$$

From (1'), when $\frac{w_0}{\gamma} = 1$ the rollover decision is identical to that when $w_0 = 1$ and $\gamma = 1$ in Table 1 and a creditor makes his rollover decision directly without involving a fund manager.

For a given $x^*$, we consider the position of the bank. When the fundamental value of the bank is $\theta$, the proportion of its creditors calling is $Pr(\theta + \delta\epsilon^j < x^*|\theta) = H(\frac{x^*-\theta}{\delta})$. Therefore, the bank with marginal fundamental value $\widehat{\theta}$ has a proportion $H(\frac{x^*-\widehat{\theta}}{\delta})$ of its creditors calling. By the definition of $\widehat{\theta}$ and the nature of creditor runs, we have

$$c = F \cdot H(\frac{x^* - \widehat{\theta}}{\delta}). \tag{2}$$

The system of equations (1)-(2) determines the creditor-run equilibrium.[17] We have Lemma 1.

[16] In the benchmark (for purpose of comparison), we assume that the residual cash in a bank at $T_2$ carried over from $T_1$, if any, is not contractible (e.g., the equityholder can steal it). This assumption is temporary and will not be necessary in the full equilibrium with an interbank market, where in equilibrium no banks have residual cash.

[17] Now we can specify $x^{*U}$ as a function of $\theta^U$, which is given by $\int_{\theta^U}^{\infty} (\gamma R) \cdot \pi(\theta) \cdot h_g(\theta|x^{*U})d\theta = w_0$. Hence, to ensure that when $\theta \geq \theta^U$, the bank can always satisfy its creditors' early withdrawals on its own, the equilibrium $x^*$ must satisfy the condition of $c \geq F \cdot H(\frac{x^*-\theta^U}{\delta})$.

**Lemma 1** *Without an interbank market, the creditor-run equilibrium at $T_1$ is characterized by the pair $(x^*, \widehat{\theta})$, which solves the system of equations (1)-(2) for a given $c$ and $R$. The equilibrium is unique when $\frac{\delta}{\sigma}$ is small enough (the equilibrium is stable).*[18] *We have comparative statics $\frac{\partial x^*}{\partial c} < 0$ and $\frac{\partial x^*}{\partial R} < 0$.*

***Proof.*** *See Appendix C.* ■

The intuition behind the comparative statics in Lemma 1 is as follows. If the bank has more cash holdings, it can withstand withdrawals by more creditors and is less vulnerable, thereby lowering the threshold for coordination among creditors to not run. In other words, a higher $c$ means that the chance of the accident of miscoordination among creditors at $T_1$ is lower; consequently, creditors are more comfortable with staying until $T_2$ and thus set a lower running threshold $x^*$. A higher $R$ means that creditors have a higher stake in the bank at $T_2$ and thus have lower incentives to run. Throughout the paper, we change $\delta$ while keeping $\sigma$ constant, and hence the statement that "$\frac{\delta}{\sigma}$ is small enough" means that "$\delta$ is small enough".

It is instructive to examine the creditor-run equilibrium under the limit $\delta \to 0$ for a given $\sigma$. By (2), we have $\widehat{\theta} = x^* - \delta\Phi^{-1}(\frac{c}{F})$. So we can combine (1) and (2):

$$\int_{\theta=x^*-\delta\Phi^{-1}(\frac{c}{F})}^{\theta=+\infty} (\gamma R) \cdot \pi(\theta) \cdot h_g(\theta|x^*) d\theta = w_0.$$

Under the limit $\delta \to 0$ for a given $\sigma$, it is easy to show that the above equation can be transformed to

$$\underbrace{\frac{c}{F}}_{\text{Coordination (illiquidity) risk}} \cdot \underbrace{R \cdot \pi(x^*)}_{\text{Fundamental (insolvency) risk}} = \frac{w_0}{\gamma}. \tag{3}$$

(3) becomes very intuitive: the term $\frac{c}{F}$ measures coordination (illiquidity) risk while $R \cdot \pi(x^*)$ corresponds to fundamental (insolvency) risk. The uniqueness of the solution of $x^*$ is straightforward. The economic intuition behind the derivation of (3) is the following. Under the limit $\delta \to 0$, fundamental uncertainty disappears, i.e., $\theta \to x^*$. However, strategic uncertainty does not. From the marginal creditor's perspective, the proportion of creditors calling loans is uniformly distributed

[18] In a "stable" equilibrium, the best response function (of an individual player to its peers) intersects the $45^0$ line at a slope of less than 1. See, e.g., Morris and Shin (2003), Vives (2005, 2014) and Angeletos et al. (2006, 2007).

within $[0,1]$. So the probability that the proportion of creditors calling loans is less than $\frac{c}{F}$ is exactly $\frac{c}{F}$.

**Remark 1** *Our benchmark model can be regarded as a hybrid of the models of Rochet and Vives (2004) (Vives (2014)), Goldstein and Pauzner (2005), Morris and Shin (2009) and Liu and Mello (2011). We have chosen a model setting tailored to our purpose to study creditor runs in the presence of an interbank market. Morris and Shin (2009) and Vives (2014) provide a comprehensive analysis on illiquidity risk and insolvency risk for a single financial institution.*

### 3.2 Equilibrium at $T_1$ with an interbank lending market

Without an interbank lending market at $T_1$, the cash holdings of a bank are either not enough or wasted at $T_1$. Now we consider the case where an interbank lending market opens at $T_1$, so banks can borrow and lend funds among themselves.

The formal definition of the equilibrium at $T_1$ with an interbank market is as follows.

**Definition 1** *An equilibrium at $T_1$ is defined by a triplet $(x^*, \widehat{\theta}, I)$ for a given $c$ and $R$, where $x^*$ is the rollover threshold for creditors of a bank, $\widehat{\theta}$ denotes the marginal bank in the system that survives at $T_1$, and $I$ is the risk-adjusted gross interbank market rate, such that (i) given rational expectations of $I$, creditors of a bank set their optimal rollover threshold as $x^*$; and (ii) given creditors' strategy $x^*$ in all banks, the competitive interbank market determines $(\widehat{\theta}, I)$.*

The equilibrium in fact comprises two elements: the creditor-run equilibrium for an *individual* bank and the interbank-market equilibrium. We analyze them one by one.

**The creditor run for an individual bank in equilibrium** Like the autarky case, the creditor-run equilibrium for *individual* bank $i$ is characterized by $(x^{*i}, \widehat{\theta}^i)$, where $x^{*i}$ is the rollover threshold of creditors of bank $i$ and $\widehat{\theta}^i$ is the failure threshold for bank $i$. Superscript '$i$' in notations $\widehat{\theta}^i$ and $x^{*i}$ highlights the fact that creditors of each individual bank are with 'local thinking' — they only consider the position of their own bank and do not necessarily have a global view of the banking system.

The creditor-run equilibrium for bank $i$ is given by

$$\int_{-\infty}^{\widehat{\theta}^{i}} 0 \cdot h_g(\theta|x^{*i})d\theta + \int_{\widehat{\theta}^{i}}^{\infty} (R \cdot \pi(\theta)) \cdot h_g(\theta|x^{*i})d\theta = \frac{w_0}{\gamma} \tag{4a}$$

$$c + \frac{\mathbb{C}(\widehat{\theta}^{i}; x^{*i})}{I} = F \cdot H(\frac{x^{*i} - \widehat{\theta}^{i}}{\delta}), \tag{4b}$$

where function $\mathbb{C}(\theta; x^{*i}) = \left[(1-c)X - R \cdot F \cdot \left(1 - H(\frac{x^{*i}-\theta}{\delta})\right)\right] \pi(\theta)$ denotes the *collateral value* of bank $i$ with asset quality $\theta$. (4a)-(4b) parallel (1)-(2). The difference is that the presence of an interbank market allows an individual bank to borrow liquidity from it, so we have (4b), in comparison with (2). In fact, if the borrowing is impossible (i.e., $I = +\infty$), (4b) exactly corresponds to (2). Under the competitive interbank market, every bank is a price-taker and takes the interbank rate $I$ as given. The liquidity status of the bank depends on its own liquidity holdings, $c$, and the short-term funds it can raise from the interbank market. (4b) gives the condition under which the bank will fail for a given $I$.

We explain (4b). When the bank has asset quality $\theta$, a number of $F \cdot H(\frac{x^{*i}-\theta}{\delta})$ creditors will call it at $T_1$, and the remaining $F \cdot \left(1 - H(\frac{x^{*i}-\theta}{\delta})\right)$ creditors will stay until $T_2$. Hence, the expected payoff that will accrue to the bank's equityholder at $T_2$, after paying its staying creditors, is $\mathbb{C}(\theta; x^{*i}) = \left[(1-c)X - R \cdot F \cdot \left(1 - H(\frac{x^{*i}-\theta}{\delta})\right)\right] \pi(\theta)$, which is the collateral value. Figure 4 shows the balance sheet position for the bank at $T_1$. So the bank can borrow a maximum $\frac{\mathbb{C}(\theta;x^{*i})}{I}$ from the interbank market at $T_1$; that is, $c_1^T(\theta) = c + \frac{\mathbb{C}(\theta;x^{*i})}{I}$. Because its required liquidity is $F \cdot H(\frac{x^{*i}-\theta}{\delta})$, we can obtain the bank failure threshold $\widehat{\theta}^{i}$.

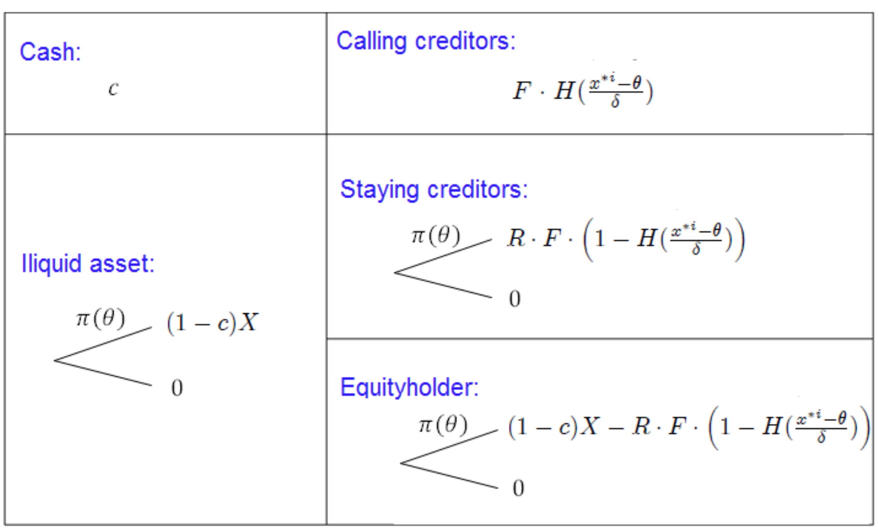


**Figure 4**: Bank $i$'s balance sheet position at $T_1$

The following lemma summarizes the creditor-run equilibrium for an individual bank.

**Lemma 2** *In the presence of an interbank market, the creditor-run equilibrium for individual bank $i$ is characterized by the pair $(x^{*i}, \widehat{\theta}^i)$, which solves the system of equations (4a)-(4b) for a given expectation of $I$. When the equilibrium is stable, we have comparative statics $\frac{\partial x^{*i}}{\partial I} > 0$.*

***Proof.*** *See Appendix C.* ■

Lemma 2 highlights the coordination equilibrium within a bank. The intuition behind the comparative statics in Lemma 2 is as follows. As in Lemma 1, the liquidity status of a bank impacts the *coordination risk* among its creditors in debt rollover decisions. *Ceteris paribus,* a rise in funding cost in the interbank market means a deterioration of a bank's liquidity status. So the coordination risk among creditors increases and thus creditors run more often. That is, the expectation of an increase in $I$ leads to a higher $x^{*i}$.[19]

**Remark 2** *Rochet and Vives (2004) study a creditor-run model with global games for a single bank in the presence of an exogenous interbank market. They show that the creditor-run equilibrium is affected by the interbank rate (i.e., parameter $\lambda$ in their model). Our model so far (Lemma 2) resembles Rochet and Vives (2004). The interbank rate $I$ in our model parallels parameter $\lambda$ in their model. In what follows, we endognize the interbank market and the interbank rate $I$.*[20,21]

**The interbank market in equilibrium** Given creditors' strategy $x^*$ in all banks $\ell \neq i$, the interbank market in equilibrium is given by the following two joint equations:

$$\int_{\widehat{\theta}}^{\infty} cg(\theta;\mu)d\theta = \int_{\widehat{\theta}}^{\infty} F \cdot H(\frac{x^* - \theta}{\delta})g(\theta;\mu)d\theta \tag{5a}$$

$$I = \frac{\mathbb{C}(\widehat{\theta}; x^*)}{F \cdot H(\frac{x^* - \widehat{\theta}}{\delta}) - c}. \tag{5b}$$

[19] A higher interbank rate leads to greater *downside risk* for bank creditors, but no upside risk. Concretely, creditors of a bank suffer if their bank cannot borrow from the interbank market but they do not benefit if their bank makes a profit from interbank lending (i.e., all the profits accrue to the bank equityholder), by noting that the profits are in terms of cash flow on top of $X$ due to the positive asset payoff correlation at $T_2$ across banks. So creditors react to their expectation of a higher interbank rate by running more often.

[20] Rochet and Vives (2004) provide two alternative interpretations for parameter $\lambda$: asymmetric information and liquidity problems. Our study is along the line of their second interpretation — liquidity problems.

[21] As will be shown, in our model, both the demand and supply of liquidity in the interbank market are endogenous. So there does not exist an aggregate demand curve (function) of bank assets such that the amount of liquidity a bank can raise is a function of the quantity of its assets put up for sale or put forward as collateral.

(5a) is the interbank market clearing condition, where $\widehat{\theta}$ denotes the marginal bank in the system that survives at $T_1$. Banks indexed $\theta \geq \widehat{\theta}$ survive while all others fail. Hence, the aggregate liquidity that surviving banks possess is $\int_{\widehat{\theta}}^{\infty} cg(\theta;\mu)d\theta$. For a bank with asset quality $\theta$, its required liquidity for survival is $Pr(\theta + \delta\epsilon^j < x^*|\theta) = F \cdot H(\frac{x^*-\theta}{\delta})$. So market clearing dictates that equation (5a) must be true. (5b) says that in equilibrium the interbank rate $I$ must be equal to the *marginal* bank $\widehat{\theta}'$s collateral value divided by its funding shortage. This is due to the nature of the *competitive* interbank market. In fact, if the price deviates from the one in (5b), the interbank market will not clear. Concretely, if the interbank rate is higher, the marginal bank $\widehat{\theta}$ as well as the banks of a lower quality would not be able to afford the liquidity, and so the total supply of liquidity would exceed the total demand. On the other hand, if the interbank rate is lower, some banks of a lower quality would be able to afford and thus would demand and *compete* for liquidity, and so the total demand would exceed the total supply.

More concretely, based on (5a)-(5b), three segments of banks exist endogenously at $T_1$ according to their realized asset quality. Let $\theta^T$ be the solution to $c = F \cdot H(\frac{x^*-\theta^T}{\delta})$.

*Failing Banks* $\theta \in \left[-\infty, \widehat{\theta}\right)$ These banks do not participate in the interbank market (i.e., they neither lend nor borrow). They do not borrow because they cannot afford the term $I$. In fact, if such a bank were to borrow at the rate $I$, the borrowed liquidity plus its own liquidity would still be insufficient to cover the demand of its creditors, i.e., $c + \frac{\mathbb{C}(\theta;x^*)}{I} < F \cdot H(\frac{x^*-\theta}{\delta})$. The liquidity shortage means that a bank run will occur, and the bank's long-term project will terminate prematurely and be *physically* liquidated (with $\varepsilon \to 0$ liquidation value),[22] so no lenders are willing to lend to it in the first place. We also assume that a bank that goes bankrupt at $T_1$ has no ability at $T_2$ to recover its investment if it lends in the interbank market, so it does not lend in the first place.[23]

*Borrowing Banks* $\theta \in \left[\widehat{\theta}, \theta^T\right)$ These banks do not have enough of their own liquidity to cover their creditors' withdrawals. To avoid bankruptcy, such a bank will borrow just enough to cover its creditors' withdrawals. Bank $\theta$'s required liquidity is $F \cdot H(\frac{x^*-\theta}{\delta})$, so it will borrow $F \cdot H(\frac{x^*-\theta}{\delta}) - c$.

[22] A bank run makes the ex post Coasian negotiation impossible (Diamond and Rajan (2001)).

[23] When a decision makes no difference to its equity value, a bank will choose to maximize the value of its debtholders.

*Lending Banks* $\theta \in \left[\theta^T, \infty\right]$ These banks have excess liquidity. As long as $I \geq 1$, such a bank will have incentives to lend out its excess liquidity. Thus, the net borrowing is $F \cdot H(\frac{x^*-\theta}{\delta}) - c$, which is negative. It will be shown that the optimal amount of cash holdings decided at $T_0$ in equilibrium cannot be "excess", so that $I \geq 1$ is true at $T_1$.

From the above, equations (5a)-(5b) constitute a *competitive equilibrium* because: i) given the interbank rate $I$ and $x^*$, banks make their optimal borrowing and lending decisions; ii) the borrowing and lending of banks clear the interbank market. Put slightly differently, the price given by (5b) is the only price that can clear the market. If $I$ deviates, then $\widehat{\theta}$ will be different based on (5b), so equation (5a) cannot be satisfied. We summarize the interbank-market equilibrium as follows.

**Lemma 3** *The competitive equilibrium of the interbank market is characterized by $(\widehat{\theta}, I)$, which is the unique solution to equations (5a)-(5b) for a given $x^*$ (under a small enough $c$, precisely established in the proof). We have $\frac{\partial I}{\partial x^*} > 0$ and $\frac{\partial I}{\partial \mu} < 0$.*

***Proof.*** *See Appendix C.* ■

The intuition for Lemma 3 is the following. The more conservative the creditors are (i.e., a higher $x^*$), the more liquidity a bank needs (i.e., a higher $F \cdot H(\frac{x^*-\theta}{\delta})$ for every given $\theta$). Banks will demand more and supply less liquidity in the interbank market, and hence the equilibrium interbank rate $I$ will be higher. When there is an aggregate negative shock to asset quality (i.e., a lower $\mu$, by recalling $\theta \sim N(\mu, \sigma^2)$), $\theta$ will *in general* become smaller and some banks will need more liquidity. This means that either the aggregate demand for liquidity in the interbank market will increase or the aggregate supply will decrease or both, which drives up the interbank rate.

Finally, because of the *symmetric* equilibrium of creditors' strategy across banks, we have

$$x^{*i} = x^*. \tag{6}$$

We summarize the equilibrium at $T_1$ in Proposition 1.

**Proposition 1** *In the presence of an interbank market, the creditor run-interbank market equilibrium at $T_1$ is defined by the triplet $(x^*, \widehat{\theta}, I)$, which solves the system of equations (4a)-(4b), (5a)-(5b) and (6) for a given $c$ and $R$.*

Proposition 1 highlights the interdependence of runs on financial institutions and trade in the interbank market. Earlier works such as Bhattacharya and Gale (1987) and Allen and Gale (2000) model the interbank market based on the assumption that there exist exogenous ex post bank-specific liquidity shocks. In our model, the illiquidity risk of banks is endogenous, originating in bank-specific solvency shocks. Using our framework of the interbank market, we show that the rollover decision of creditors of an individual bank and the interbank rate are jointly determined in equilibrium, which in turn is dependent on the status of the aggregate economy (fundamentals $\mu$). These results, not already known in the literature, help explain the evidence and facts discussed in the introduction.

The equilibrium at $T_1$ in fact comprises two subequilibria. We have the following corollary.

**Corollary 1** *The (within-bank) creditor-run equilibrium ((4a)-(4b) and (6)) determines $x^*$ for each expectation of $I$, while the (cross-bank) interbank-market equilibrium ((5a)-(5b)) determines $I$ for a given $x^*$. The creditor run-interbank market equilibrium in the system at $T_1$ is characterized by the fixed point problem between $x^*(I)$ and $I(x^*;\mu)$.*

***Proof.*** *See Appendix C.* ■

### 3.3 Characterization of the equilibrium

We will now characterize the equilibrium in Proposition 1. In our model, there is two-way feedback between $x^*$ and $I$. In general, two-way feedback can generate multiple equilibria. We first characterize the existence of a unique equilibrium.

**Proposition 2** *The creditor run-interbank market equilibrium at $T_1$ is unique when $\frac{\delta}{\sigma}$ is small enough (the equilibrium is stable).*

***Proof.*** *See Appendix C.* ■

We can find the unique equilibrium in Proposition 2 under the limit $\delta \to 0$ for a given $\sigma$. In this limiting case, we have $R \cdot \pi(x^*) = \frac{w_0}{\gamma}$, $\Phi(\frac{\widehat{\theta}-\mu}{\sigma}) = \frac{F \cdot \Phi(\frac{x^*-\mu}{\sigma})-c}{F-c}$ and $I = \frac{[(1-c)X]\cdot\pi(\widehat{\theta})}{F-c}$.

When $\frac{\delta}{\sigma}$ is high enough, equilibrium uniqueness may not hold. In this case, a changed expectation of creditors on the interbank rate will change the creditor-run equilibrium, which in turn can result in an interbank-market equilibrium that actually fulfills the different interbank rate conjectured. That is, there may exist multiple *self-fulfilling* creditor run-interbank market equilibria.

**Proposition 3** *When $\frac{\delta}{\sigma}$ is high enough, there are multiple (typically two) creditor run-interbank market equilibria. One equilibrium (the higher $I$ one) is Pareto-dominated by the other equilibrium (the lower $I$ one).*

***Proof.*** *See Appendix C.* ■

Proposition 3 may explain the phenomenon of *self-fulfilling* interbank market freezes. Whereas the classic works like Diamond and Dybvig (1983) show a self-fulfilling creditor run in a bank, we show that a self-fulfilling freeze can occur in an interbank market. Intuitively, if creditors believe that the interbank rate will be high and hence that their bank will be able to raise less liquidity from the interbank market, they optimally choose to run more often. If creditors run more often, all banks rationally demand more and supply less liquidity to protect themselves, leading to an actual high interbank rate. Creditors' beliefs are thus confirmed and become self-fulfilling.[24]

When $\delta$ becomes higher (for a given $\sigma$), creditors depend more on public information to make their decisions and therefore equilibrium multiplicity becomes more likely (see also Morris and Shin (2003) and Vives (2005, 2014)).

**Illustration** *To better grasp the intuition behind equilibrium multiplicity in our model, we plot in Figure 5 the payoff function for the marginal agent, $Y(x^*)$, under a set of parameter values. Function $Y(x^*)$ is the expected payoff of rolling over for a creditor who receives a signal just equal to $x^*$ given that all other creditors within the bank as well as creditors of other banks use threshold $x^*$ (see (C.7) in Appendix C for the characterization). The solutions to equation $Y(x^*) = \frac{w_0}{\gamma}$ give the equilibria. The parameter values except $\delta$ for Figure 5 are given in Appendix B.*

[24] Our self-fulfilling equilibrium might also carry over under the alternative assumption that banks move first and creditors move later. In this case, banks' trade in the interbank market that determines $I$ depends on their *expectation* about $x^*$; after seeing the interbank rate $I$, creditors' rollover decision fulfills $x^*$ in equilibrium.

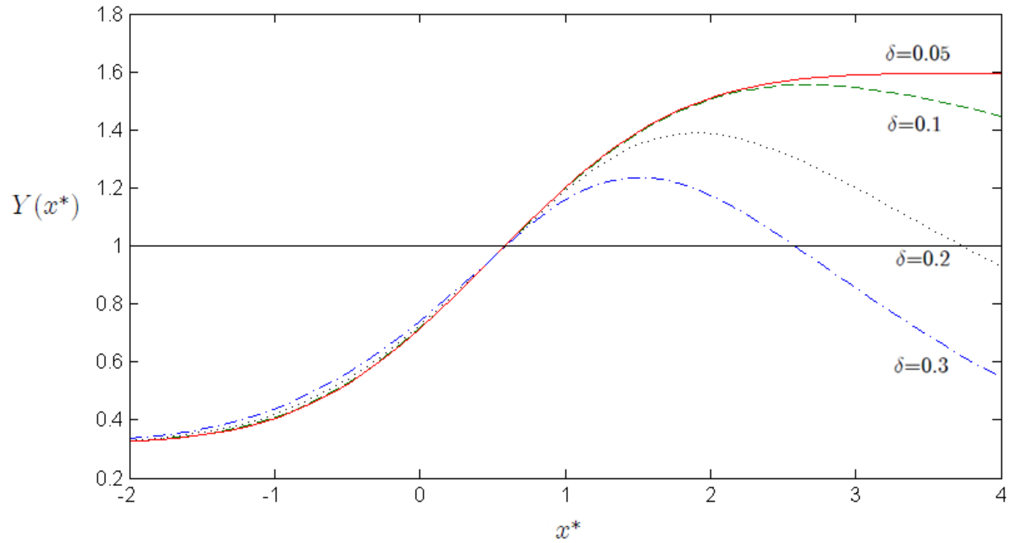


**Figure 5**: Marginal agent's payoff function $Y(x^*)$ when private precision changes

*In Figure 5, the intersections of the curves and the solid line ($\frac{w_0}{\gamma} = 1$) represent the equilibria. When $\delta$ is low enough, the equilibrium is unique. When $\delta$ is high enough, there exist two equilibria. The number of equilibria is the number of intersections that fall into the non-upper-dominance region $(-\infty, x^{*U})$.*

We denote two equilibria by $(x_L^*, \widehat{\theta}_L, I_L)$ and $(x_H^*, \widehat{\theta}_H, I_H)$, where $x_L^* < x_H^*$, $\widehat{\theta}_L < \widehat{\theta}_H$ and $I_L < I_H$. The latter (unstable) is Pareto-dominated by the former (stable). Figure 6 illustrates the two equilibria; the vertical axis measures the threshold used by an individual creditor $(x^{*j})$ while the horizontal axis measures the threshold used by other creditors within the bank as well as by creditors of other banks $(x^*)$. That is, $x^{*j}(x^*)$ is the best response function for an individual creditor.

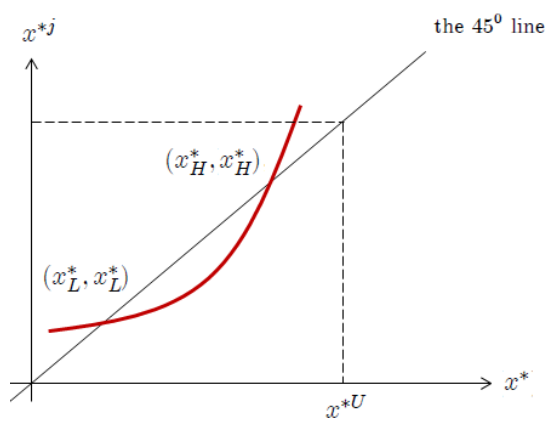


**Figure 6**: Multiple (two) equilibria at $T_1$

One corollary to Proposition 3 follows.

**Corollary 2** *For a given $\frac{\delta}{\sigma}$, equilibrium multiplicity (two equilibria) is more likely under a lower $c$ or a lower $\mu$, ceteris paribus.*

***Proof.*** *See Appendix C.* ■

The intuition behind Corollary 2 is the following. *Ceteris paribus*, a lower $c$ or a lower $\mu$ makes it more likely for two possible equilibria to both fall into the non-upper-dominance region, $(-\infty, x^{*U})$. Figure C-1 in Appendix C gives an illustration.

**Remark 3** *In Vives (2014), strategic complementarity increases with a higher liquidity ratio. This is because a supersolvency region or an upper dominance region exists in Vives (2014) but not in our model. If one considers an extended model of Vives (2014) by changing only the liquidation function in his model to allow for non-existence of a supersolvency region (for example, the liquidation value per unit in his model becomes $\frac{1}{1+\lambda}$ with constraint $M + \frac{I}{1+\lambda} < D$, instead of $\frac{\theta}{1+\lambda}$), the extended model of Vives (2014) delivers results consistent with those of our model.*[25]

## 4 Equilibrium at $T_0$

In this section, we study the equilibrium at $T_0$; in doing so we will endogenize $c$ and $R$. A key result of this section is that allocation $(c, R)$ is a function of the aggregate status of the economy at $T_1$ — distribution $\theta \sim N(\mu, \sigma^2)$ or $\mu$. This result will be useful in the analysis in the next section.

To save space, we outline the basic logic and steps in finding the equilibrium at $T_0$ while relegating the full analysis to Appendix A. We consider both the constrained second-best equilibrium and the competitive equilibrium.

**Constrained second-best equilibrium** There are two steps in finding the constrained second-best equilibrium. First, we obtain the expected payoff to a creditor and hence his ex ante participation condition at $T_0$.[26] This will determine $R$ for a given $c$. Second, we consider the social

[25] The analysis is available upon request.

[26] We assume that when a creditor decides on his participation $R$, he takes into account the fact that his fund manager's objective is not entirely aligned with his own. The participation condition without involving a fund manager needs the full continuous-state creditor-run payoff structure without simplification as in Goldstein and Pauzner (2005). See the recent work of Allen, Carletti, Goldstein and Leonello (2015) along this line.

planner's problem. The social planner's objective is to maximize the total social surplus — the aggregate value of all banks in the economy, including the failing banks at $T_1$ and the surviving banks at $T_2$. The social planner faces the following tradeoff: A higher $c$ results in more banks surviving at $T_1$ (i.e., a lower $\widehat{\theta}$), but also less investment in long-term projects in the economy. The tradeoff leads to an optimal liquidity level $c$ at $T_0$.

**Competitive equilibrium** Our main focus in this paper is on the frictions at $T_1$, i.e., information asymmetry about banks' status at $T_1$, so we abstract away from other frictions. Specifically, we assume that the amount of cash holdings in banks $c$ is publicly observed (and contractible) at $T_0$, as in the way Freixas and Rochet (2008) (p. 233-234) treat the model of Bhattacharya and Gale (1987). Thus, the optimal cash holdings in the constrained optimum can be written in the private contract between banks. That is, the constrained optimum can be implemented in the competitive equilibrium because the contractibility solves the commitment issue. Alternatively, we can assume that there is an ex-ante liquidity regulation in place, so all banks hold cash ex ante based on the regulatory rule. Either way, the competitive equilibrium at $T_0$, in which banks decide on their ex ante liquidity holdings based on individual rationality, coincides with the constrained second-best equilibrium.[27]

## 5 Aggregate shock: crisis and amplification

In this section, we study our model under aggregate uncertainty, which has implications for the occurrence of a crisis and amplification.

So far we have assumed that there is no aggregate uncertainty but only idiosyncratic shocks to banks; that is, the distribution $g(\cdot)$ is given and deterministic. Now we consider the case with aggregate uncertainty. Specifically, we assume that there are two states of nature at $T_1$ for the aggregate economy: normal state ($s = N$) and bad state ($s = B$). Ex ante, at $T_0$, the normal state will occur with probability $q$ and the bad state will occur with complementary probability. In

[27] Ahnert (2013), among others, studies the efficiency of the ex ante cash holdings. Allen et al. (2009) and Freixas et al. (2011) examine ex post central bank interventions. Brunnermeier and Oehmke (2013b) discuss various amplification mechanisms in the recent crisis. Other generally related papers investigate optimal maturity structure choices (Brunnermeier and Oehmke (2013a)), the shadow banking system (Plantin (2013)), repo runs (Martin, Skeie and von Thadden (2014)), and the interaction between two runs (Liu (2015)).

$s = N$ the asset quality distribution is given by $g_N(\cdot)$, and in $s = B$ the distribution is $g_B(\cdot)$, where $g_N$ has first-order stochastic dominance over $g_B$. Specifically, the distribution of $\theta$ is $\theta \sim N(\mu, \sigma^2)$ in state $s = N$ and $\theta \sim N(\mu_B, \sigma^2)$ in state $s = B$, where $\mu > \mu_B$.

In studying the ex-ante problem without aggregate uncertainty (Section 4 and Appendix A.1), we have shown that for a given distribution $g(\cdot)$, the equilibrium at $T_0$ determines allocation $(c, R)$. So, if agents can perfectly foresee the aggregate state, $N$ or $B$, we have the corresponding allocation decided at $T_0$, denoted by $(c_s, R_s)$, where $s = N$ and $B$. In comparison, in the presence of aggregate uncertainty, agents are uncertain about the aggregate state at $T_1$, so the ex-ante allocation $(c, R)$ is prepared for the "average" of the states $s = N$ and $B$ and $(c, R)$ is an "average" of $(c_N, R_N)$ and $(c_B, R_B)$ (see Appendix A.2).

Given $(c, R)$ set at $T_0$, the equilibrium at $T_1$ varies for different realized states $s = N$ and $B$. We write the equilibrium correspondence as

$$(c, 1-c, R) \xrightarrow{g_s} (x_s^*, \widehat{\theta}_s, I_s),$$

where $1 - c$ denotes the unit of illiquid asset, and $s = N$ and $B$. Our focus of study is on what happens when bad state $s = B$ occurs.

To sharpen the implication, we consider the extreme case: $q \to 1$. This means that the bad state will occur with *negligible probability.* Because the normal state will occur almost surely, the allocation at $T_0$ is completely determined by the distribution $g_N(\cdot)$; namely, the allocation is $(c_N, R_N)$.[28] State $s = B$ in this case can be interpreted as the economy suffering an unexpected aggregate shock (away from the normal state $s = N$). However, it is important to emphasize that our model mechanism in this section depends on the existence of aggregate uncertainty, not on the assumption that the shock is unanticipated.[29]

The equilibrium at $T_1$ under the shock (i.e., state $s = B$) is given by the system of equations (4a)-(4b), (5a)-(5b) and (6), where the allocation $(c, R)$ is $(c_N,\ R_N)$, the asset quality distribution

[28] Note that in our model, agents are risk-neutral and therefore their marginal utility is constant (across states).

[29] If the shock is anticipated (i.e., state $s = B$ with a *positive* ex ante probability), the ex-ante allocation $(c, R)$ is prepared for the "average" of the states $s = N$ and $B$. Amplification will still occur at $T_1$ in state $s = B$ because the realized state is a bad "shock" relative to the expectation of occurrence of the "average" of the states.

$g(\cdot)$ in (5a) is replaced by $g_B(\cdot)$, and the public information in (4a) is also replaced by $g_B(\cdot)$. The equilibrium outcome is intuitively denoted by $(x^*_{NB}, \widehat{\theta}_{NB}, I_{NB})$, namely, the equilibrium under allocation $(c_N,\, R_N)$ with shock $g_N \to g_B$.

To highlight the mechanism, we focus on the case in which the equilibrium for the normal state, $(x^*_N, \widehat{\theta}_N, I_N)$, is unique. In this case, when the shock hits, there are two scenarios: the new equilibrium, $(x^*_{NB}, \widehat{\theta}_{NB}, I_{NB})$, can be either unique or multiple.

We examine the first scenario, which is summarized in Proposition 4.

**Proposition 4** *Suppose the equilibrium in the normal state, $(x^*_N, \widehat{\theta}_N, I_N)$, is unique. If the shock, $\mu - \mu_B$, is small, the new equilibrium, $(x^*_{NB}, \widehat{\theta}_{NB}, I_{NB})$, is also unique. We have $x^*_{NB} > x^*_N$, $\widehat{\theta}_{NB} > \widehat{\theta}_N$ and $I_{NB} > I_N$.*

***Proof.*** *See Appendix C.* ■

The adverse shock $g_N \to g_B$ (or $\mu \downarrow \mu_B$) triggers a chain of actions and reactions in the system. The starting point of the chain is $\frac{\partial I(x^*;\mu)}{\partial \mu} < 0$; that is, given $x^*$ unchanged, a lower $\mu$ leads to a higher $I$.[30] Then, there is a feedback loop between $I$ and $x^*$, as illustrated in Figure 7. A higher interbank rate exacerbates creditor runs ($\frac{\partial x^*(I)}{\partial I} > 0$). In turn, more severe runs in financial institutions tighten the liquidity in the interbank market, driving up the interbank rate ($\frac{dI(x^*;\mu)}{dx^*} > 0$). Empirically, $I$ measures the degree of interbank market freeze and $x^*$ measures the degree of liquidity evaporation in the system. Therefore, interbank market freezes and liquidity evaporation reinforce each other, which helps explain the unprecedented events in the recent financial crisis discussed at the beginning of this paper. Put slightly differently, an adverse aggregate shock triggers the reinforcing spiral between a decreasing *supply* of liquidity (i.e., interbank market tightness) and an increasing *demand* for liquidity (i.e., creditor runs) in the system.

[30] A lower $\mu$ has another channel of effect; that is, the public information changes from $g_N$ to $g_B$ in (4a). In fact, (4a)-(4b) give $x^{*i}(I;\mu)$, where $\mu$ is public information. Intuitively, each individual creditor of a bank receives bad news about the asset quality of his own bank. In reality, the ABX index might be such public information.

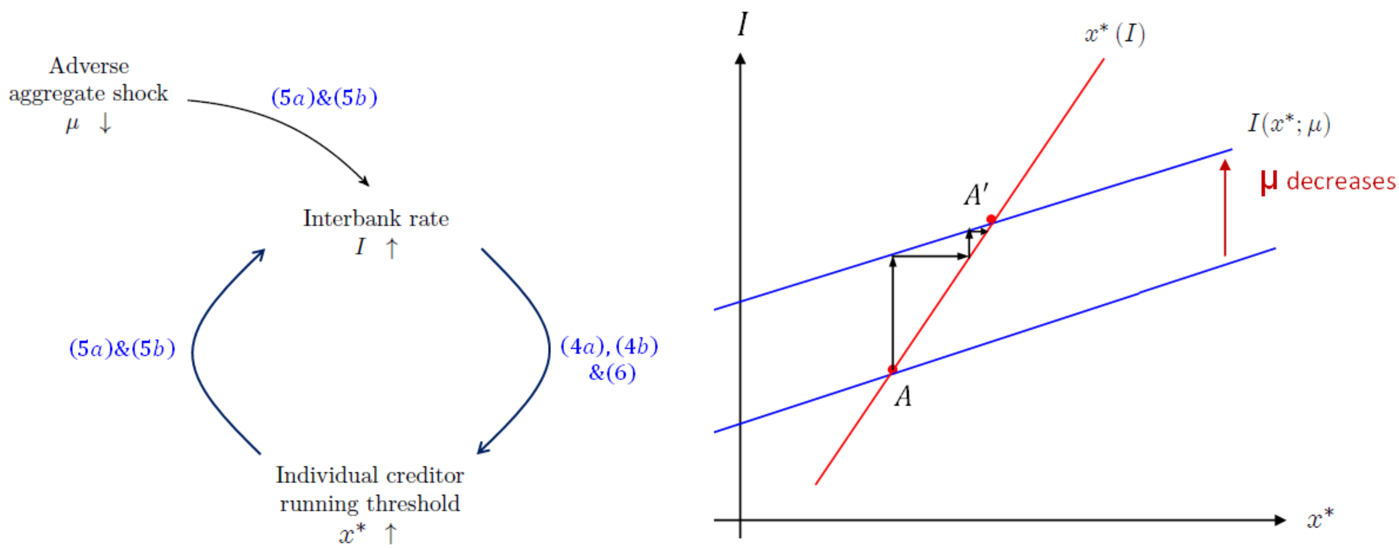


**Figure 7**: Amplification through a feedback loop and spiral along a stable equilibrium

Next, we study the second scenario, which is Proposition 5.

**Proposition 5** *Suppose the equilibrium in the normal state, $(x_N^*, \widehat{\theta}_N, I_N)$, is unique. If the shock, $\mu - \mu_B$, is large enough, multiplicity of the new equilibrium, $(x_{NB}^*, \widehat{\theta}_{NB}, I_{NB})$, can emerge. That is, amplification can also come through triggering multiple equilibria.*

***Proof.*** *See Appendix C.* ■

Proposition 5 shows another channel of amplification — triggering self-fulfilling multiple equilibria. As shown in Figure 8, a small negative shock ($\mu_B'$) can move $A$ to $A'$ (which is the case in Proposition 4 and Figure 7), whereas a large enough shock ($\mu_B''$) can move $A$ to either $A''$ or $B''$ (which is the case in Proposition 5).

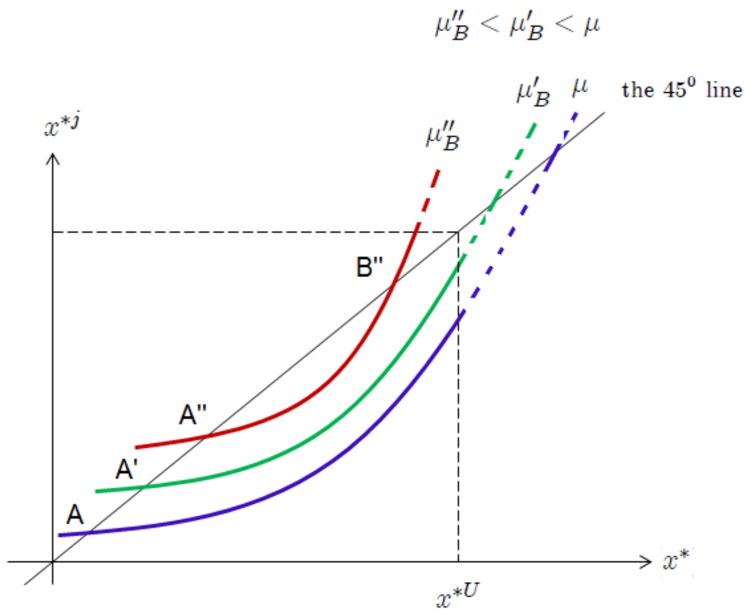


**Figure 8**: Amplification through triggering multiple equilibria

We would like to point out the limitations of the results associated to an unstable equilibrium (namely, Proposition 3 and Proposition 5). According to Vives (2005, 2014), adaptive dynamics in terms of best reply dynamics will not lead to an unstable equilibrium. The appearance of an unstable equilibrium may not mean much if the system moves back to the stable equilibrium.

**Aggregate liquidity shortage** From the above analysis, banking crises in our model occur because financial institutions tap the same pool of short-term funding. In our model, a lower-quality bank fails not because it suffers a loss in its investment and thereby its capital when investing in other banks, or because its own asset is hit by a negative shock and thus its demand for liquidity increases. Rather, the key problem can be that some higher-quality banks in the system suffer a negative shock to their asset and thus need more liquidity than expected (in order to reduce their illiquidity risk). The decrease in the net supply of liquidity from these higher-quality banks leads to shrinking liquidity in the pool, which actually hurts lower-quality banks first and foremost. In other words, negative shocks may hit higher-quality banks but it is the lower-quality banks that suffer first. In fact, the financial crisis of 2007-2009 originated in the US, where there had been a subprime mortgage crisis, yet the first bank to suffer bank runs was Northern Rock, a UK bank.[31]

[31] In the literature that studies contagion through interbank claims (e.g., Allen and Gale (2000, 2004), Freixas et al. (2000), Dasgupta (2004), Rochet and Tirole (1996), Freixas and Holthausen (2005)), the shocks are typically on

## 6 Policy implications

In this section, we discuss policy implications of our model. We analyze two ex post intervention measures when a negative aggregate shock hits.

**Liquidity injections** The analysis in Section 5 implies that a crisis goes with a shortage of aggregate liquidity. Therefore, a natural ex post intervention policy is to inject liquidity into the financial system. It is important to note that the *pure* promise of bailout in our model cannot achieve the same effect of deposit insurance in Diamond and Dybvig (1983) as a *costless* solution to liquidity crises. This is because creditor runs in our model are both fundamental- and panic-based, rather than purely panic-based (i.e., coordination problems) as in Diamond and Dybvig (1983) (see Appendix C for a formal analysis).

Actual liquidity injection is necessary in intervention**.** Specifically, we consider the following simple intervention scheme: when the bad state ($s = B$) occurs, the central bank provides liquidity assistance to every bank in the system — every bank receives an amount of liquidity, $\triangle c$, from the central bank (before creditors make their rollover decisions). Here we make a weak assumption that the central bank does not know the quality of individual banks and hence distributes the liquidity *evenly* across banks.

Program 1 gives the problem of the ex post optimal intervention policy for the central bank:

$$\begin{aligned} \max_{\triangle c \in [0,\infty)} \quad & \int_{\widehat{\theta}_{GB}(\triangle c)}^{\widehat{\theta}_{NB}} (1 - c_N)(X \cdot \pi(\theta)) g_B(\theta) d\theta + \triangle c - \tau\left(\triangle c\right) \qquad \text{(Program 1)} \\ \text{s.t.} \quad & (c_N + \triangle c, 1 - c_N, R_N) \xrightarrow{g_B} (x^*_{GB}, \widehat{\theta}_{GB}, I_{GB}) \end{aligned}$$

We explain Program 1. Without the ex post intervention, the threshold of the marginal bank surviving is $\widehat{\theta}_{NB}$. With the ex post intervention, the equilibrium at $T_1$ is summarized by $(c_N + \triangle c, 1 - c_N, R_N) \xrightarrow{g_B} (x^*_{GB}, \widehat{\theta}_{GB}, I_{GB})$, where $(x^*_{GB}, \widehat{\theta}_{GB}, I_{GB})$ denotes the equilibrium outcome under the government intervention in the state $s = B$. We have that the new threshold of the marginal bank $\widehat{\theta}_{GB}$ is a decreasing function of $\triangle c$; that is, a higher amount of liquidity injections lowers the threshold of the marginal bank in the system. In fact, liquidity injections enable banks

the liability side and contagion stems from contractual links between banks. Our paper models the shocks on the asset side and highlights propagation through the impact on the illiquidity risk rather than on the insolvency risk.

in the range $\theta \in \left[\widehat{\theta}_{GB}(\triangle c), \widehat{\theta}_{NB}\right]$ to survive; otherwise they would fail, which is the gain from the government intervention. In addition, the injected liquidity, $\triangle c$, stays in the private sector, which is the second term of the objective function in Program 1. The total cost of the intervention is $\tau(\triangle c)$, where $\tau(\cdot)$ denotes the gross (opportunity) cost function of funding (including pecuniary and non-pecuniary costs), and $\tau(\cdot)$ satisfies $\tau' > 0$, $\tau'' > 0$, and $\tau(x) \geq x$ for any $x \geq 0$. The government might source this funding through the issuance of public debt (domestic or foreign) at $T_1$, which it will repay by collecting tax from taxpayers (creditors) at $T_2$.

It is straightforward to see that in Program 1, the optimal amount of liquidity assistance, $\triangle c$, can be positive. The reason is simple: the gain from liquidity assistance, measured by the distance of $\widehat{\theta}_{NB} - \widehat{\theta}_{GB}(\triangle c)$, can be high. Indeed, based on Corollary 2, the increase in cash can eliminate self-fulfilling multiple equilibria besides breaking the feedback cycle; both effects reduce $\widehat{\theta}$. Remark 4 immediately follows.

**Remark 4** *When the bad state ($s = B$) occurs, an optimal ex post intervention of liquidity injections is given by Program 1. Liquidity injections help not only to break the feedback spiral along the stable equilibrium but also to eliminate self-fulfilling multiple equilibria.*

Two comments are in order. First, the efficiency gain from the intervention comes from saving illiquid projects from premature liquidation. Without intervention, the aggregate liquidity shortage in the system would lead to the early closure and physical liquidation of illiquid long-term projects in banks indexed $\theta \in \left[\widehat{\theta}_{GB}(\triangle c), \widehat{\theta}_{NB}\right]$. Second, when the economy is hit by an aggregate shock, the purpose of government intervention is not to save a single bank, but to influence the interaction (among banks and with creditors) within the system and thereby improve overall efficiency. In fact, as we have shown, a small aggregate shock can trigger the amplification mechanisms that lead to aggregate liquidity shortage of the system, with the marginal bank threshold rising from $\widehat{\theta}_N$ to $\widehat{\theta}_{NB}$, which justifies the ex post intervention. Regulators do not need to identify individual vulnerable banks. They can simply offer liquidity assistance to all banks and the interbank market will play a role of reallocating liquidity. This is different from the situation where the negative shock is idiosyncratic, in which case regulators face the classic problem of information asymmetry in deciding whether or not to bail out or give liquidity assistance to a certain bank — whether that bank is illiquid or insolvent (see, e.g., Rochet and Vives (2004) and Brunnermeier et al. (2009)).

In the recent crisis, the Federal Reserve created emergency liquidity facilities (e.g., the Asset-Backed Commercial Paper Money Market Mutual Fund Liquidity Facility). Moreover, in the U.S., the Federal Reserve adopted an unconventional policy of credit easing through a combination of lending to financial institutions, providing liquidity directly to key credit markets, and purchasing long-term securities (Bernanke (2009)). Central banks in Europe and Japan used similar quantitative easing policies.

**Public disclosure** Suppose that the central bank is informed of the asset quality of individual banks in the system. The information may come from the supervisory knowledge of the central bank as in Rochet and Vives (2004) or from crisis-time stress tests conducted by the central bank as in Goldstein and Leitner (2013). We consider a simple public disclosure rule: the central bank classifies and discloses to the market a bank's type — type A or type B.[32] We show that there is a unique optimal percentage of banks that the central bank should classify and disclose as type-A banks in maximizing the aggregate bank value of the system.

Formally, after the public disclosure, each creditor of a bank has two pieces of information based on which to make his rollover decision: public information (his bank's type) and private signal $x^{ij}$. Public information alters the prior distribution of $\theta$ and thus the posterior conditional density of $\theta$. Concretely, suppose that the central bank discloses $\lambda$ proportion of banks as type-A banks. Then, a creditor of a type-A bank knows that his bank's asset quality must satisfy $\theta \geq \underline{\theta}$, where $\Phi(\frac{\underline{\theta}-\mu_B}{\sigma}) = 1-\lambda$. Thus, for the marginal creditor of a type-A bank, his posterior conditional density $h_g(\theta|x^{*i})$ in (4a) is replaced by

$$\hat{h}_g(\theta|x^{*i}) = \frac{\frac{1}{\sigma}\phi(\frac{\theta-\mu_B}{\sigma})\frac{1}{\delta}\phi(\frac{x^{*i}-\theta}{\delta})}{\int\limits_{\theta=\underline{\theta}}^{\theta=+\infty} \frac{1}{\sigma}\phi(\frac{\theta-\mu_B}{\sigma})\frac{1}{\delta}\phi(\frac{x^{*i}-\theta}{\delta})d\theta} \text{ for } \theta \geq \underline{\theta}. \tag{7}$$

That is, $\hat{h}_g(\theta|x^{*i})$ is a *truncated* conditional normal distribution due to the public disclosure. In addition, it is easy to see that an optimal disclosure rule must be such that in equilibrium all type-B

[32] Considering only two types is without loss of generality. Considering more types would not change the model result. In fact, the optimal disclosure rule that maximizes the number of banks surviving at $T_1$ must be such that in equilibrium all banks classified as the highest ranking type survive while all other types of banks fail; so non-surviving banks in equilibrium can belong to one type (e.g., "B" only) or more than one type (e.g., "B", "C",...), which does not matter.

banks fail while none or a fraction of type-A banks whose asset quality is in the lower range fail. Hence, the following condition is true:

$$\underline{\theta} \leq \widehat{\theta}. \tag{8}$$

The equilibrium at $T_1$ under the shock (i.e., state $s = B$) with public disclosure is given by the system of equations (4a)-(4b), (5a)-(5b) and (6) with constraint (8), where the allocation $(c, R)$ is $(c_N,\ R_N)$, the asset quality distribution $g(\cdot)$ in (5a) is replaced by $g_B(\cdot)$, and the posterior conditional density $h_g(\theta|x^{*i})$ in (4a) is replaced by (7). We have the following proposition.

**Proposition 6** *Public disclosure at $T_1$ can increase the aggregate bank value of the system. There exists a unique optimal $\lambda \in (0, 1)$ in public disclosure.*

***Proof.*** *See Appendix.* ■

Public disclosure can improve welfare. The intuition is the following. Without public disclosure, a creditor knows that his bank's asset quality follows the prior distribution $\theta \sim N(\mu_B, \sigma^2)$. After public disclosure, a creditor of a type-A bank knows that his bank's asset quality belongs to the top $\lambda$ (percentage) of the distribution $\theta \sim N(\mu_B, \sigma^2)$. In this sense, a creditor of such a bank has received good news because his bank is "upgraded" (intuitively speaking, mean $\mu_B$ is improved). Therefore, creditors of a type-A bank run less often (i.e., a lower $x^*$) and consequently the marginal bank threshold $\widehat{\theta}$ decreases; that is, the number of banks surviving at $T_1$ increases. More concretely, when $\lambda$ decreases (i.e., fewer type-A banks), the type-A threshold ($\underline{\theta}$) increases while the marginal bank threshold ($\widehat{\theta}$) decreases. There is a unique $\lambda$ such that $\underline{\theta}$ and $\widehat{\theta}$ coincide, at which the aggregate bank value is maximized.

The intuition for the existence of a unique optimal $\lambda$ is the following. If the government classifies and discloses too many banks as type-A banks, the disclosure itself would not be very informative; creditors of a type-A bank will still run very often and thus there will still be a lot of banks failing (among type-A banks). In the extreme case where $\lambda = 1$, public disclosure would not change $\widehat{\theta}$ in equilibrium at all and thus is useless. In contrast, if the government discloses too few type-A banks, creditors of such a type-A bank will feel very secure and will be less likely to run; however, type-A banks are far and few between in the market in the first place. Therefore, it is not optimal for the government to disclose too many or too few type-A banks. In other words, there is a unique

$\lambda$ that leads to an optimal degree of "pooling" to maximize the total number of banks surviving in the system.

Our analysis of public disclosure intervention in this subsection is along the line of the work of Goldstein and Leitner (2013), Vives (2014) and Bouvard et al. (2015), but offers new insights. Our analysis highlights the impact of public disclosure on creditor runs in the presence of an interbank market.

# 7 Conclusion

This paper presents a model of interbank lending that helps explain a systemic crisis. On the methodology side, we study global games in general equilibrium, so that we can explicitly model a *competitive* interbank market, and examine how the interaction among creditors within a bank affects and is in turn affected by other banks through the interbank market.

Our model demonstrates banking crises originating in fundamental shocks (i.e., aggregate shocks to banks' asset quality) and shows the interplay between illiquidity risk and insolvency risk in a financial system context. The paper highlights how the feedback between runs on financial institutions and trade in the interbank market can amplify a small shock into a systemic crisis. A crisis propagates in our model because institutions rely on short-term funding from the same pool. We also examine how the ex post intervention measures of liquidity injections and public disclosure can improve efficiency.

# Appendix

## A Equilibrium at $T_0$

### A. 1 Equilibrium at $T_0$ without aggregate uncertainty

In this section, we study the equilibrium at $T_0$; in doing so we will endogenize $c$ and $R$. Since banks are identical at $T_0$, to reduce notational clutter, we drop the bank index superscript $i$ in this section, unless otherwise specified.

We study the constrained second-best equilibrium. We state the equilibrium concept.

**Constrained second-best equilibrium** A constrained second-best equilibrium consists of the following three elements:

(i) For a given $(c, R)$ set at $T_0$, the equilibrium outcome at $T_1$ is $(x^*, \widehat{\theta}, I)$.[33]

(ii) A creditor demands an interest rate $R$ at $T_0$ such that given $(c, R)$ and the subsequent equilibrium outcome $(x^*, \widehat{\theta}, I)$, the creditor breaks even ex ante at $T_0$.

(iii) Knowing the response of $(R, x^*, \widehat{\theta}, I)$ to $c$, the social planner chooses an optimal $c$ at $T_0$ to maximize the aggregate expected value of all banks in the economy.

**Step 1: How $R$ is determined**

We work out the *ex post* payoff to a creditor in the creditor-run game. If a bank with fundamentals $\theta$ fails at $T_1$, the bank's total asset is the cash $c$, which is divided among the creditors who call. Hence, a creditor who calls obtains $\frac{c}{F \cdot H(\frac{x^*-\theta}{\delta})}$ by recalling that the proportion of creditors calling is $H(\frac{x^*-\theta}{\delta})$, and a creditor who does not call obtains 0. If a bank survives to $T_2$, a number of $F \cdot H(\frac{x^*-\theta}{\delta})$ creditors has called at $T_1$, each of whom has obtained face value 1, while each staying creditor's expected payoff is $\pi(\theta)R$ at $T_2$.[34]

Thus, we can obtain the expected payoff to a creditor and thereby his ex ante participation

[33] If there are multiple equilibria of $(x^*, \widehat{\theta}, I)$ ex post at $T_1$, agents anticipate that the efficient (stable) equilibrium will be selected. This is equivalent to there being an ex post regulatory intervention in place.

[34] Staying creditors make their claim on the $1-c$ units of the long-term risky asset. We will show that in equilibrium $(1-c)X > FR$. So a staying creditor obtains $R$ if and only if the long-term asset realizes its high state payoff $X$.

condition at $T_0$. That is,

$$\begin{aligned}R_0 &= \int_{-\infty}^{\widehat{\theta}} \left[ \underbrace{\int_{-\infty}^{x^*} \frac{c}{F \cdot H(\frac{x^*-\theta^i}{\delta})} \cdot h(x^{ij}|\theta^i) dx^{ij}}_{\text{Call on failing bank}} + \underbrace{\int_{x^*}^{+\infty} 0 \cdot h(x^{ij}|\theta^i) dx^{ij}}_{\text{Hold on failing bank}} \right] g(\theta^i) d\theta^i \\ &+ \int_{\widehat{\theta}}^{\infty} \left[ \underbrace{\int_{-\infty}^{x^*} 1 \cdot h(x^{ij}|\theta^i) dx^{ij}}_{\text{Call on surviving bank}} + \underbrace{\int_{x^*}^{+\infty} \big(R \cdot \pi(\theta^i)\big) \cdot h(x^{ij}|\theta^i) dx^{ij}}_{\text{Hold on surviving bank}} \right] g(\theta^i) d\theta^i. \qquad \text{(A.1)}\end{aligned}$$

(A.1) gives the position for creditor $j$ in bank $i$. Ex ante, at $T_0$, this creditor faces two levels of uncertainty: the quality of his bank and the signal that he will receive. The density $g(\theta^i)$ represents the first level of uncertainty and the conditional density $h(x^{ij}|\theta^i)$ represents the second level of uncertainty. The payoff in each scenario was explained in the previous paragraph.[35]

Equation (A.1) can be simplified and rewritten as

$$R_0 = \int_{-\infty}^{\widehat{\theta}} (\frac{c}{F}) \cdot g(\theta) d\theta + \int_{\widehat{\theta}}^{\infty} \left[ H(\frac{x^*-\theta}{\delta}) \cdot 1 + \left(1 - H(\frac{x^*-\theta}{\delta})\right) \cdot (R \cdot \pi(\theta)) \right] g(\theta) d\theta. \qquad \text{(A.1')}$$

(A.1') is very intuitive. Conditional on a bank's failing, the expected payoff to a creditor is $\frac{c}{F}$. This is because $c$ will be distributed among the calling creditors while ex ante all creditors have an equal chance of ending up as calling creditors; therefore, ex ante, this is equivalent to $c$ being distributed equally among a number of $F$ creditors. Conditional on the bank's surviving, with probability $H(\frac{x^*-\theta}{\delta})$, a creditor receives a bad signal and thus calls, in which case his payoff is 1; with probability $1 - H(\frac{x^*-\theta}{\delta})$, he receives a good signal and thus stays, in which case his expected payoff is $R \cdot \pi(\theta)$.

For cleanness, we might explicitly add another constraint:

$$FR < (1-c)X. \qquad \text{(A.2)}$$

Constraint (A.2) gives a sufficient condition to guarantee that creditors are repaid with $R$ (no default) when the long-term asset realizes its high-state cash flow $X$ at $T_2$. This condition, however,

[35] The four terms in (A.1) correspond to the four elements in Table 1. The difference in payoffs between Table 1 (under $w_0 = 1$, $\gamma = 1$ and no involvement of a fund manager) and (A.1) is minor. The participation condition (A.1) is in terms of the *gross* payoff to a creditor. The idea is that when a creditor decides on his participation $R$, he takes into account that his fund manager's objective is not entirely aligned with his own. In particular, under the assumption that $w_0 \to 0$ and $\gamma \to 0$, the net payoff to a creditor approaches his gross payoff.

is not necessary in general when the optimization problem, to be shown shortly, is taken into account. That is, it is not optimal in equilibrium to choose too high a $c$.

**Step 2: How $c$ is determined**

Now we study the decision of the social planner at $T_0$, i.e., element (iii). The social planner's objective is to maximize the total social surplus. The social planner's problem is

$$\begin{aligned} \max_{c} \quad & \int_{-\infty}^{\widehat{\theta}} c g(\theta) d\theta + \int_{\widehat{\theta}}^{\infty} \left[c + (1-c)(X \cdot \pi(\theta))\right] g(\theta) d\theta \qquad \text{(Program A1)} \\ \text{s.t.} \quad & \text{(4a)-(4b), (5a)-(5b), (6), (A.1') and (A.2)} \end{aligned}$$

In Program A1, the objective function is to maximize the aggregate value of all banks in the economy, including the failing banks at $T_1$ (the first term) and surviving banks at $T_2$ (the second term). This aggregate value, in the end, is divided between the equityholders and creditors in the economy (see the proof of Lemma 4 in Appendix C).[36] Lemma 4 summarizes the result.

**Lemma 4** *The constrained second-best equilibrium solves Program A1, which gives ex ante (c,R). The equilibrium exists.*

***Proof.*** *See Appendix C.* ∎

We explain the intuition behind the optimal $c$ in Lemma 4. A higher $c$ results in more banks surviving at $T_1$ (i.e., a lower $\widehat{\theta}$), but also less investment in long-term projects in the economy. The tradeoff leads to an optimal liquidity ratio at $T_0$. Specifically, denoting the aggregate bank value by $V$ in the objective function in Program A1, the first-order derivative of $V$ is

$$\frac{\partial V}{\partial c} = \underbrace{\left\{ \left(-\frac{\partial \widehat{\theta}}{\partial c}\right) \cdot \left[(1-c)(X \cdot \pi(\widehat{\theta}))g(\widehat{\theta})\right] \right\}}_{\text{more banks survive ex post}} - \underbrace{\left[\int_{\widehat{\theta}}^{\infty} (X \cdot \pi(\theta))g(\theta)d\theta - 1\right]}_{\text{higher return of banks ex ante}}.$$

The deadweight loss when a bank of quality $\theta$ fails at $T_1$ is $(1-c)(X \cdot \pi(\theta))$, so the gain from having more banks survive in the economy by holding one more unit of cash at $T_0$ is $(-\frac{\partial \widehat{\theta}}{\partial c}) \cdot \left[(1-c)(X \cdot \pi(\widehat{\theta}))g(\widehat{\theta})\right]$. On the other hand, storing cash for banks means that valuable investment opportunities are wasted in the economy, with the loss being $\int_{\widehat{\theta}}^{\infty} (X \cdot \pi(\theta))g(\theta)d\theta - 1$. The tradeoff

[36] Program A1 is equivalent to maximizing the aggregate equity value. The equivalence is because creditors of a bank, in total, claim a *constant residual* value, $FR_0$.

leads to an optimal level of cash holdings for banks at $T_0$. Moreover, we show that under the sufficient condition that $\underline{\pi}$ is small enough, the optimal $c$, as determined in Program A1, is such that some banks will fail at $T_1$ (i.e., $\widehat{\theta} > -\infty$).

## A. 2 Equilibrium at $T_0$ with aggregate uncertainty

For a given $(c, R)$ set at $T_0$, the equilibrium at $T_1$ under distribution $g_s$ is written as

$$(c, 1-c, R) \xrightarrow{g_s} (\widehat{\theta}_s, x_s^*, I_s). \tag{A.3}$$

With the two aggregate states at $T_1$ (i.e., $s = N$ and $B$), a creditor's ex-ante participation condition at $T_0$ is given by

$$R_0 = \sum_{s=N,B} q_s \left\{ \int_{-\infty}^{\widehat{\theta}_s} (\frac{c}{F}) g_s(\theta) d\theta + \int_{\widehat{\theta}_s}^{\infty} \left[ H(\frac{x_s^* - \theta}{\delta}) \cdot 1 + \left(1 - H(\frac{x_s^* - \theta}{\delta})\right) (R \cdot \pi(\theta)) \right] g_s(\theta) d\theta \right\} \tag{A.4}$$

and hence the constrained second-best equilibrium at $T_0$ is given by

$$\begin{array}{lll} \max\limits_{c} & \sum\limits_{s=N,B} q_s \left[ \int_{-\infty}^{\widehat{\theta}_s} c g_s(\theta) d\theta + \int_{\widehat{\theta}_s}^{\infty} \left[c + (1-c)(X \cdot \pi(\theta))\right] g_s(\theta) d\theta \right] & \\ \text{s.t.} & \text{(A.2), (A.3) and (A.4)} & \text{(Program A2)} \end{array}$$

where $q_N = q$ and $q_B = 1 - q$.

**Lemma 5** *With aggregate uncertainty, the constrained second-best equilibrium solves Program A2, which gives ex ante (c,R). The equilibrium exists. When $q \to 1$, $(c, R) \to (c_N, R_N)$; when $q \to 0$, $(c, R) \to (c_B, R_B)$.*

***Proof.*** *See Appendix C.* ■

Intuitively, agents are uncertain about the aggregate state at $T_1$, so the ex-ante allocation $(c, R)$ is prepared for the "average" of the states of $s = N$ and $B$. When the bad state ($s = B$) is rather unlikely (i.e., $1-q$ is low), the ex ante allocation in equilibrium is largely based on the consideration of the occurrence of state $s = N$.

# B Numerical example

We provide a numerical example to illustrate the main results of the model. The numerical example is to illustrate the qualitative (rather than quantitative) aspect of the model.

We set the parameter values as follows: $X = 8$, $F = 0.3$, $R_0 = 1.03$, $\frac{w_0}{\gamma} = 1$, $\underline{\pi} = 0.2$, $\overline{\pi} = 1$, $\mu = 0.5$, $\sigma = 1$, $\delta = 0.2$, $\mu_\pi = 0.5$, $\sigma_\pi = 1$. We assume that $\theta^U = 2.4$, which corresponds to $\pi(\theta^U) = 97.7\%$. The endogenous variables in the model are $(c, R, x^*, \widehat{\theta}, I)$.

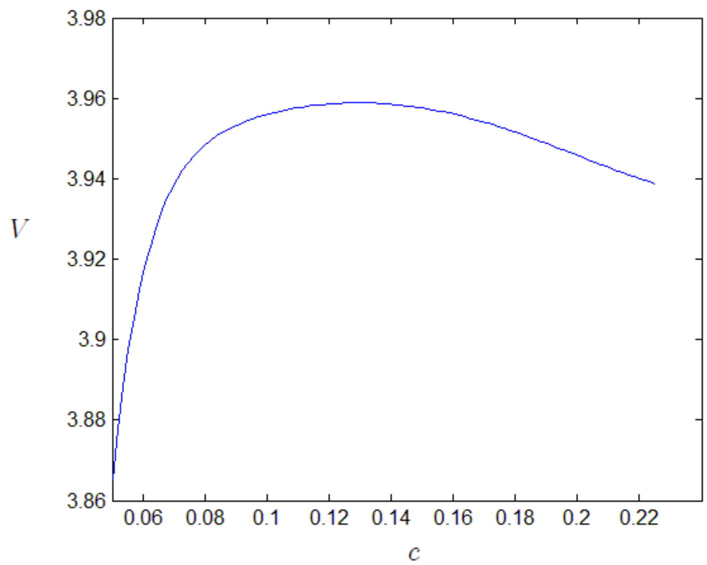


**Figure B-1**: Aggregate bank value in the second-best equilibrium

*The second-best equilibrium at $T_0$* The aggregate bank value $V$ in Lemma 4 is a '∩'-shaped function, as shown in Figure B-1. The optimal amount of cash holdings is $c = 0.1287$, at which $V = 3.9588$, and $R = 1.5961$. Given the $R$, the upper dominance region for the creditor-run equilibrium at $T_1$ is $[x^{*U}, +\infty) = [2.547, +\infty)$.

*The creditor run-interbank market equilibrium at $T_1$* Under $c = 0.1287$ and $R = 1.5961$ set at $T_0$, we can find a unique equilibrium at $T_1$ (Proposition 2) in which $x^* = 0.5881$, $\widehat{\theta} = -0.3986$ or $\pi(\widehat{\theta}) = 34.75\%$, and $I = 14.145$. Note that $x^* \in (-\infty, x^{*U}]$ and condition $c \geq F \cdot H(\frac{x^* - \theta^U}{\delta})$ is satisfied.

*Unique equilibrium at $T_1$ under a shock* Suppose the shock, $\mu - \mu_B$, is small. Specifically, let $\mu_B = -0.5$. The new equilibrium is still unique (Proposition 4), in which $x^*_{NB} = 0.6974$, $\widehat{\theta}_{NB} = 0.3085$ or $\pi(\widehat{\theta}_{NB}) = 53.93\%$, and $I_{NB} = 22.945$. Note that $x^* \in (-\infty, x^{*U}]$ and condition $c \geq F \cdot H(\frac{x^* - \theta^U}{\delta})$ is satisfied.

*Multiple (two) equilibria at $T_1$ under a shock* Suppose the shock, $\mu - \mu_B$, is not small. Specifically, let $\mu_B = -1$. The new equilibrium is multiple (Proposition 5). For the lower (efficient) equilibrium, $x^*_{NB} = 0.98$, $\widehat{\theta}_{NB} = 0.7032$ or $\pi(\widehat{\theta}_{NB}) = 66.44\%$, and $I_{NB} = 31.4597$. For the higher (inefficient) equilibrium, $x^*_{NB} = 2.0340$, $\widehat{\theta}_{NB} = 1.8359$ or $\pi(\widehat{\theta}_{NB}) = 92.74\%$, and $I_{NB} = 51.9671$. For both equilibria, $x^* \in (-\infty, x^{*U}]$ and condition $c \geq F \cdot H(\frac{x^*-\theta^U}{\delta})$ is satisfied.

# C Proofs

**Proof of Lemma 1:** The prior density of $\theta$ is $g(\theta)$ with distribution $N(\mu, \sigma^2)$. The signal is $x^* = \theta + \delta\epsilon$, where $\epsilon \sim N(0,1)$. So, the posterior density $h_g(\theta|x^*)$ is with distribution $N(\frac{\alpha}{\alpha+\beta}\mu + \frac{\beta}{\alpha+\beta}x^*, \frac{1}{\alpha+\beta})$, where $\alpha = \frac{1}{\sigma^2}$ and $\beta = \frac{1}{\delta^2}$. Let $\mu_\theta = \frac{\alpha}{\alpha+\beta}\mu + \frac{\beta}{\alpha+\beta}x^*$ and $\sigma_\theta = \sqrt{\frac{1}{\alpha+\beta}}$. Then, $h_g(\theta|x^*) = \frac{1}{\sigma_\theta}\phi\left(\frac{\theta-\mu_\theta}{\sigma_\theta}\right)$.

By (2), we have $\widehat{\theta} = x^* - \delta\Phi^{-1}(\frac{c}{F})$. So we can combine (1) and (2):

$$\int_{\theta=x^*-\delta\Phi^{-1}(\frac{c}{F})}^{\theta=+\infty} R\cdot\pi(\theta)\cdot\frac{1}{\sqrt{\frac{1}{\alpha+\beta}}}\phi\left(\frac{\theta-\left(\frac{\alpha}{\alpha+\beta}\mu+\frac{\beta}{\alpha+\beta}x^*\right)}{\sqrt{\frac{1}{\alpha+\beta}}}\right)d\theta = \frac{w_0}{\gamma}. \tag{C.1}$$

Write the LHS of (C.1) as function $Y(x^*;\delta)$.

The key to the proof is to determine the monotonicity of $Y(x^*;\delta)$ with respect to $x^*$. Three forces are at work in determining the monotonicity: i) the lower boundary of the integral, $x^* - \delta\Phi^{-1}(\frac{c}{F})$, is increasing in $x^*$, so $Y(x^*;\delta)$ tends to be decreasing in $x^*$; ii) the conditional mean, $\frac{\alpha}{\alpha+\beta}\mu + \frac{\beta}{\alpha+\beta}x^*$, is increasing in $x^*$, so $Y(x^*;\delta)$ tends to be increasing in $x^*$; iii) $\pi(\theta)$ is an increasing function, which makes $Y(x^*;\delta)$ tend to be increasing in $x^*$.

We conduct a transformation on $Y(x^*;\delta)$:

$$\begin{aligned}
Y(x^*;\delta) &= \int_{\theta=x^*-\delta\Phi^{-1}(\frac{c}{F})}^{\theta=+\infty} R\cdot\pi(\theta)\cdot\frac{1}{\sigma_\theta}\phi\left(\frac{\theta-\mu_\theta}{\sigma_\theta}\right)d\theta \\
&= \int_{z=\frac{x^*-\delta\Phi^{-1}(\frac{c}{F})-\mu_\theta}{\sigma_\theta}}^{z=+\infty} R\cdot\pi(\mu_\theta+\sigma_\theta z)\cdot\phi(z)\,dz \quad \text{(changing variables to } z = \frac{\theta-\mu_\theta}{\sigma_\theta}) \\
&= \int_{z=\frac{\frac{\alpha}{\alpha+\beta}(x^*-\mu)-\delta\Phi^{-1}(\frac{c}{F})}{\sqrt{\frac{1}{\alpha+\beta}}}}^{z=+\infty} R\cdot\pi\left((\frac{\alpha}{\alpha+\beta}\mu+\frac{\beta}{\alpha+\beta}x^*)+\sqrt{\frac{1}{\alpha+\beta}}z\right)\cdot\phi(z)\,dz. \qquad (C.2)
\end{aligned}$$

1) We consider the limiting case of $\delta \to 0$ for a given $\sigma$. In this case, $\frac{\beta}{\alpha+\beta} \to 1$. Hence, in (C.1), the conditional mean, $\frac{\alpha}{\alpha+\beta}\mu + \frac{\beta}{\alpha+\beta}x^*$, and the lower boundary of the integral, $x^* - \delta\Phi^{-1}(\frac{c}{F})$, increase at the *same* speed. So $Y(x^*;\delta)$ is certainly increasing in $x^*$ since $\pi(\cdot)$ is an increasing function. Concretely, under the limit $\delta \to 0$ for a given $\sigma$, we have:

$$\begin{aligned}
\lim_{\delta\to 0} Y(x^*;\delta) &= \lim_{\delta\to 0}\left( \int_{z=\frac{\frac{\alpha}{\alpha+\beta}(x^*-\mu)-\delta\Phi^{-1}(\frac{c}{F})}{\sqrt{\frac{1}{\alpha+\beta}}}}^{z=+\infty} R\cdot\pi\left( (\frac{\alpha}{\alpha+\beta}\mu + \frac{\beta}{\alpha+\beta}x^*) + \sqrt{\frac{1}{\alpha+\beta}}z \right)\cdot\phi(z)\,dz \right) \\
&= R\cdot\pi(x^*)\cdot \int_{z=-\Phi^{-1}(\frac{c}{F})}^{z=+\infty} \phi(z)\,dz \\
&= R\cdot\pi(x^*)\cdot\frac{c}{F}.
\end{aligned}$$

Thus (3) is obtained. (C.1) admits a unique solution with respect to $x^*$ (under proper parametric values of $c$, $F$ and $R$ to make the research question interesting) because $Y(x^*;\delta)$ is monotonically increasing in $x^*$. Note that $\lim_{\delta\to 0} Y(x^*;\delta) = R\cdot\pi(x^*)\cdot\frac{c}{F}$ *pointwise*, not globally uniformly.

2) We consider the non-limiting case. We prove that when $\delta$ is sufficiently small for a given $\sigma$, (C.1) also admits a unique solution with respect to $x^*$ in the interval $x^* \in (-\infty, x^{*U}]$. Note that the assumed upper dominance region implies $x^* \notin (x^{*U}, +\infty)$. The proof has three steps.

First, we prove the following result: for *any* given bounded (finite) interval around $x^* = \mu_\pi$, denoted by $[\mu_\pi - \underline{l}, \mu_\pi + \bar{l}]$, there exists a sufficiently small $\rho > 0$ such that when $\delta < \rho$, function $Y(x^*;\delta)$ is monotonically increasing in the interval $x^* \in [\mu_\pi - \underline{l}, \mu_\pi + \bar{l}]$. Concretely, based on (C.2), the first-order derivative of $Y(x^*;\delta)$ implies

$$\begin{aligned}
\frac{\partial Y(x^*;\delta)}{\partial x^*}/R &= \underbrace{-\left[\underline{\pi} + (\overline{\pi}-\underline{\pi})\Phi(\frac{x^*-\delta\Phi^{-1}(\frac{c}{F})-\mu_\pi}{\sigma_\pi})\right]\cdot\phi\left(\frac{\frac{\alpha}{\alpha+\beta}(x^*-\mu)-\delta\Phi^{-1}(\frac{c}{F})}{\sqrt{\frac{1}{\alpha+\beta}}}\right)\cdot\frac{\frac{\alpha}{\alpha+\beta}}{\sqrt{\frac{1}{\alpha+\beta}}}}_{<0} \\
&\quad \underbrace{+ \int_{z=\frac{\frac{\alpha}{\alpha+\beta}(x^*-\mu)-\delta\Phi^{-1}(\frac{c}{F})}{\sqrt{\frac{1}{\alpha+\beta}}}}^{z=+\infty} (\overline{\pi}-\underline{\pi})\phi\left(\frac{\left(\frac{\alpha}{\alpha+\beta}\mu + \frac{\beta}{\alpha+\beta}x^*\right) + \sqrt{\frac{1}{\alpha+\beta}}z - \mu_\pi}{\sigma_\pi}\right)\frac{\frac{\beta}{\alpha+\beta}}{\sigma_\pi}\cdot\phi(z)\,dz}_{>0}\ .
\end{aligned} \tag{C.3}$$

Note that when $\delta \to 0$, it follows that $\frac{\alpha}{\alpha+\beta} \to 0$, $\frac{\beta}{\alpha+\beta} \to 1$, $\frac{\delta}{\sqrt{\frac{1}{\alpha+\beta}}} \to 1$ and $\frac{\frac{\alpha}{\alpha+\beta}}{\sqrt{\frac{1}{\alpha+\beta}}} \to 0$. Hence, the first term in (C.3) converges *uniformly* to 0 in any given bounded (finite) interval $x^* \in [\mu_\pi - \underline{l}, \mu_\pi + \bar{l}]$ when $\delta \to 0$. In fact, in the first term in (C.3), $\frac{\frac{\alpha}{\alpha+\beta}}{\sqrt{\frac{1}{\alpha+\beta}}} \to 0$ when $\delta \to 0$, while $\phi(\frac{\frac{\alpha}{\alpha+\beta}(x^*-\mu)-\delta\Phi^{-1}(\frac{c}{F})}{\sqrt{\frac{1}{\alpha+\beta}}})$ is bounded and approaches $\phi(-\Phi^{-1}(\frac{c}{F}))$ when $\delta \to 0$. The second term in (C.3) is bounded from below by a *positive* value for any given finite interval $x^* \in [\mu_\pi - \underline{l}, \mu_\pi + \bar{l}]$. Therefore, we can find a sufficiently small $\rho > 0$ to make sure that when $\delta < \rho$ the first term of $\frac{\partial Y(x^*;\delta)}{\partial x^*}$ in (C.3) is sufficiently close to 0 so that the second term of $\frac{\partial Y(x^*;\delta)}{\partial x^*}$ dominates the first term. That is, $\frac{\partial Y(x^*;\delta)}{\partial x^*} > 0$.

Second, let the arbitrarily chosen interval $[\mu_\pi - \underline{l}, \mu_\pi + \bar{l}]$ be $[\underline{L}, x^{*U}]$, where $\underline{L}$ is low (such as $\underline{L} = \mu_\pi - 10\sigma_\pi$). Based on the result in the first step, there exists a sufficiently small $\rho > 0$ such that when $\delta < \rho$, equation $Y(x^*;\delta) = \frac{w_0}{\gamma}$ has a unique solution with respect to $x^*$ in $x^* \in [\underline{L}, x^{*U}]$. This is because $Y(x^*;\delta)$ is monotonically increasing in $x^*$.

Third, for the chosen $\underline{L}$ in the second step, we show that under a sufficiently small $\delta$, equation $Y(x^*;\delta) = \frac{w_0}{\gamma}$ has no solutions in the interval $x^* \in (-\infty, \underline{L})$. In fact, we can define an upper bound function of $Y(x^*;\delta)$, denoted by $\overline{Y}(x^*;\delta)$:

$$\overline{Y}(x^*;\delta) \equiv \int_{\theta=-\infty}^{\theta=+\infty} R \cdot \pi(\theta) \cdot \frac{1}{\sqrt{\frac{1}{\alpha+\beta}}} \phi\left(\frac{\theta - \left(\frac{\alpha}{\alpha+\beta}\mu + \frac{\beta}{\alpha+\beta}x^*\right)}{\sqrt{\frac{1}{\alpha+\beta}}}\right) d\theta.$$

$\overline{Y}(x^*;\delta)$ is increasing in $x^*$. Hence, in the interval $x^* \in (-\infty, \underline{L}]$, it follows that $Y(x^*;\delta) \leq \overline{Y}(x^*;\delta) \leq \overline{Y}(\underline{L};\delta)$. Considering that $\overline{Y}(\underline{L};\delta)$ is close to being $R \cdot \pi(\underline{L})$ when $\delta$ is small and is thus close to being $R\underline{\pi}$ , equation $Y(x^*;\delta) = \frac{w_0}{\gamma}$ has no solutions.

3) We prove that the unique equilibrium is also a stable equilibrium. In a stable equilibrium, the best response function (of an individual creditor to its peers) intersects the $45^0$ line at a slope of less than 1. Let an individual creditor's threshold be $x^{*j}$ and that of his peers be $x^*$. Then

$$\int_{\theta=x^*-\delta\Phi^{-1}(\frac{c}{F})}^{\theta=+\infty} R \cdot \pi(\theta) \cdot \frac{1}{\sqrt{\frac{1}{\alpha+\beta}}} \phi\left(\frac{\theta - \left(\frac{\alpha}{\alpha+\beta}\mu + \frac{\beta}{\alpha+\beta}x^{*j}\right)}{\sqrt{\frac{1}{\alpha+\beta}}}\right) d\theta = \frac{w_0}{\gamma}.$$

Write the LHS as function $Y(x^*, x^{*j})$. Applying the implicit function theorem, we have $\frac{\partial x^{*j}}{\partial x^*} = -\frac{\partial Y/\partial x^*}{\partial Y/\partial x^{*j}}$. Based on the results in 1) and 2), $Y$ is an increasing function around the solution; that is, $Y(x^* + \triangle, x^{*j} + \triangle) - Y(x^*, x^{*j}) > 0$ for a small positive $\triangle$. So $(\frac{\partial Y}{\partial x^*} + \frac{\partial Y}{\partial x^{*j}})\triangle > 0$, and thus $\frac{\partial Y}{\partial x^{*j}} > -\frac{\partial Y}{\partial x^*} > 0$. Therefore, $\frac{\partial x^{*j}}{\partial x^*} < 1$.

4) We prove the comparative statics $\frac{\partial x^*}{\partial c} < 0$ and $\frac{\partial x^*}{\partial R} < 0$. Write the LHS of (C.1) as function $Y(x^*; c, R)$. Clearly, $\frac{\partial Y}{\partial c} > 0$ while $\frac{\partial Y}{\partial x^*} > 0$. So $\frac{\partial x^*}{\partial c} = -\frac{\partial Y/\partial c}{\partial Y/\partial x^*} < 0$. Similarly, $\frac{\partial x^*}{\partial R} < 0$.

**Proof of Lemma 3:** We first prove Lemma 3 and Proposition 2 then prove Lemma 2 and Corollary 1.

1) We analyze (5a), which can be rewritten as

$$c = \frac{\int_{\widehat{\theta}}^{\infty} F \cdot \Phi(\frac{x^*-\theta}{\delta}) \frac{1}{\sigma}\phi(\frac{\theta-\mu}{\sigma}) d\theta}{\int_{\widehat{\theta}}^{\infty} \frac{1}{\sigma}\phi(\frac{\theta-\mu}{\sigma}) d\theta}. \tag{C.4}$$

Write the RHS of (C.4) as a function with respect to $\widehat{\theta}$ parameterized by $x^*$, denoted by $f(\widehat{\theta}; x^*)$. The first-order derivative of $f(\widehat{\theta}; x^*)$ implies

$$\begin{aligned} sgn\left(\frac{\partial f}{\partial \widehat{\theta}}\right) &= sgn\left(\int_{\theta=\widehat{\theta}}^{\infty} \left[\Phi(\frac{x^*-\theta}{\delta}) - \Phi(\frac{x^*-\widehat{\theta}}{\delta})\right] \frac{1}{\sigma}\phi(\frac{\theta-\mu}{\sigma}) d\theta\right) \\ &= -1, \end{aligned}$$

where the second equality is because $\Phi(\frac{x^*-\theta}{\delta})$ is decreasing in $\theta$. So $f(\widehat{\theta}; x^*)$ is decreasing in $\widehat{\theta}$ (for a given $x^*$), and it peaks at $\widehat{\theta} = -\infty$. Therefore, when $c \leq f(\widehat{\theta} = -\infty; x^*)$, equation (5a) has a unique solution with respect to $\widehat{\theta}$ for a given $x^*$. Denote the solution by $\widehat{\theta}(x^*; \mu, c)$, which is clearly decreasing in $c$. In fact,

$$\frac{\partial f}{\partial \widehat{\theta}} < 0 \Longrightarrow \frac{\partial \widehat{\theta}}{\partial c} < 0.$$

2) We prove the comparative statics $\frac{\partial \widehat{\theta}}{\partial \mu} < 0$ and $\frac{\partial \widehat{\theta}}{\partial x^*} > 1$ for solution $\widehat{\theta}(x^*; \mu, c)$ given by (5a). We proceed in three steps.

**Step 1**: Define function $f(t; \mu, x^*) \equiv \frac{\int_{\theta=t}^{\infty} F \cdot \Phi(\frac{x^*-\theta}{\delta}) \frac{1}{\sigma}\phi(\frac{\theta-\mu}{\sigma}) d\theta}{\int_{\theta=t}^{\infty} \frac{1}{\sigma}\phi(\frac{\theta-\mu}{\sigma}) d\theta}$. Transform it to

$$\begin{aligned} f(t; \mu, x^*) &= \frac{\int_{z=0}^{\infty} F \cdot \Phi(\frac{x^*-(t+z)}{\delta}) \frac{1}{\sigma}\phi(\frac{(t+z)-\mu}{\sigma}) dz}{\int_{z=0}^{\infty} \frac{1}{\sigma}\phi(\frac{(t+z)-\mu}{\sigma}) dz} \quad \text{(changing variables to } z = \theta - t) \\ &= \int_{z=0}^{\infty} F \cdot \Phi(\frac{x^*-(t+z)}{\delta}) \frac{\frac{1}{\sigma}\phi(\frac{(t+z)-\mu}{\sigma})}{1-\Phi\left(\frac{t-\mu}{\sigma}\right)} dz. \end{aligned}$$

Note that $\frac{\frac{1}{\sigma}\phi(\frac{(t+z)-\mu}{\sigma})}{1-\Phi\left(\frac{t-\mu}{\sigma}\right)} = d\frac{\Phi(\frac{t-\mu}{\sigma}+\frac{z}{\sigma})}{1-\Phi\left(\frac{t-\mu}{\sigma}\right)}/dz$ is a truncated normal density function with respect to $z$.

**Step 2**: We prove the following general property for the truncated normal density: for the truncated normal density $\varphi(t;a) = \frac{\phi(a+t)}{1-\Phi(a)}$ with $t \geq 0$, distribution $\varphi(t;a')$ has first-order stochastic dominance over distribution $\varphi(t;a'')$ for $a' < a''$. To see this, denote the c.d.f by $\Psi(t;a) = \frac{\Phi(a+t)-\Phi(a)}{1-\Phi(a)}$. We need to prove that $\frac{\partial\Psi(t;a)}{\partial a} > 0$ for every $t$. This is true because

$$\frac{\partial\Psi(t;a)}{\partial a} = \left[\frac{\phi\left(a+t\right)}{1-\Phi\left(a+t\right)} - \frac{\phi(a)}{1-\Phi(a)}\right]\frac{\left[1-\Phi\left(a+t\right)\right]}{1-\Phi(a)} > 0,$$

where the inequality follows by the monotone hazard rate (MHR) of the normal distribution.

**Step 3**: Letting $t = \widehat{\theta}$, equation (C.4) becomes $f\left(\widehat{\theta};\mu,x^*\right) = c$. By the implicit function theorem,

$$\begin{aligned}
\frac{\partial\widehat{\theta}}{\partial x^*} &= -\frac{\partial f}{\partial x^*}\Big/\frac{\partial f}{\partial\widehat{\theta}} \\
&= \frac{\displaystyle\int_{z=0}^{\infty} F\cdot\tfrac{1}{\delta}\phi\Big(\tfrac{x^*-\left(\widehat{\theta}+z\right)}{\delta}\Big)\frac{\frac{1}{\sigma}\phi\Big(\frac{\left(\widehat{\theta}+z\right)-\mu}{\sigma}\Big)}{1-\Phi\left(\frac{\widehat{\theta}-\mu}{\sigma}\right)}dz}{\displaystyle\int_{z=0}^{\infty} F\cdot\tfrac{1}{\delta}\phi\Big(\tfrac{x^*-\left(\widehat{\theta}+z\right)}{\delta}\Big)\frac{\frac{1}{\sigma}\phi\Big(\frac{\left(\widehat{\theta}+z\right)-\mu}{\sigma}\Big)}{1-\Phi\left(\frac{\widehat{\theta}-\mu}{\sigma}\right)}dz - \int_{z=0}^{\infty} F\cdot\Phi\Big(\tfrac{x^*-\left(\widehat{\theta}+z\right)}{\delta}\Big)\frac{\partial\frac{\frac{1}{\sigma}\phi\left(\frac{\left(\widehat{\theta}+z\right)-\mu}{\sigma}\right)}{1-\Phi\left(\frac{\widehat{\theta}-\mu}{\sigma}\right)}}{\partial\widehat{\theta}}dz} \\
&> 1.
\end{aligned}$$

The denominator in the second line is positive by $\frac{\partial f}{\partial\widehat{\theta}} < 0$ shown in 1). Its second term is also positive. This is because function $\Phi(\frac{x^*-\left(\widehat{\theta}+z\right)}{\delta})$ is decreasing in $z$ while density function $\frac{\frac{1}{\sigma}\phi\left(\frac{\left(\widehat{\theta}+z\right)-\mu}{\sigma}\right)}{1-\Phi\left(\frac{\widehat{\theta}-\mu}{\sigma}\right)}$ with respect to $z$ has the property of first-order stochastic dominance when $\widehat{\theta}$ decreases as shown in Step 2. So we obtain the result of $\frac{\partial\widehat{\theta}}{\partial x^*} > 1$. Similarly, we can confirm that

$$\frac{\partial\widehat{\theta}}{\partial\mu} = -\frac{\partial f}{\partial\mu}\Big/\frac{\partial f}{\partial\widehat{\theta}} = 1 - \frac{\partial\widehat{\theta}}{\partial x^*} < 0.$$

3) We analyze (5b). Since (5a) gives a unique $\widehat{\theta}$ for a given $x^*$, then $I$ is unique by (5b). We conduct comparative statics on $I$. Denote $H(\frac{x^*-\widehat{\theta}}{\delta})$ by $y$ in (5b). So (5b) can be written as

$$I = \frac{(1-c)X - R\cdot F\cdot(1-y)}{F\cdot y - c}\pi(\widehat{\theta}). \tag{C.5}$$

Clearly, in (C.5), $\frac{\partial I}{\partial\widehat{\theta}} > 0$. We also prove that $\frac{\partial I}{\partial y} < 0$. In fact,

$$\frac{\partial I}{\partial y} = F\frac{[FR-(1-c)X]-Rc}{(F\cdot y - c)^2}\pi(\widehat{\theta}) < 0,$$

where the inequality is due to (A.2).

Combining (5a) and (5b), we show the overall comparative statics $\frac{\partial I}{\partial x^*} > 0$ and $\frac{\partial I}{\partial \mu} < 0$. Consider first $\frac{\partial I}{\partial x^*} > 0$. An increase in $x^*$ leads to $\widehat{\theta}$ increasing and $y$ decreasing because $\frac{\partial \widehat{\theta}}{\partial x^*} > 1$ in (5a), which form two forces in (C.5) that drive up $I$. Now consider $\frac{\partial I}{\partial \mu} < 0$. A decrease in $\mu$ leads to $\widehat{\theta}$ increasing in (5a), so in (C.5) $\widehat{\theta}$ increases and $y$ decreases; both forces drive up $I$.

**Proof of Proposition 2:** Because of the symmetric equilibrium ($x^{*i} = x^*$) and because there is a continuum of i.i.d. banks in the system, in equilibrium the following is true:

$$\widehat{\theta} = \widehat{\theta}^i.$$

So (4a) becomes

$$\int_{\widehat{\theta}}^{\infty} (R \cdot \pi(\theta)) \cdot h_g(\theta|x^*) d\theta = \frac{w_0}{\gamma}. \tag{C.6}$$

(5a) gives a unique $\widehat{\theta}$ for a given $x^*$ (see the proof of Lemma 3); denote the solution by $\widehat{\theta}(x^*; \sigma, \delta)$. Hence, (C.6) becomes

$$\int\limits_{\theta=\widehat{\theta}(x^*;\sigma,\delta)}^{\theta=\infty} R \cdot \pi(\theta) \cdot \frac{1}{\sqrt{\frac{1}{\alpha+\beta}}} \phi \left( \frac{\theta - \left( \frac{\alpha}{\alpha+\beta}\mu + \frac{\beta}{\alpha+\beta} x^* \right)}{\sqrt{\frac{1}{\alpha+\beta}}} \right) d\theta = \frac{w_0}{\gamma}. \tag{C.7}$$

Write the LHS of (C.7) as function $Y(x^*; \sigma, \delta)$. Equation (5b) is rewritten as

$$I = \frac{\left[ (1-c)X - R \cdot F \cdot \left( 1 - H(\frac{x^* - \widehat{\theta}}{\delta}) \right) \right] \pi(\widehat{\theta})}{F \cdot H(\frac{x^* - \widehat{\theta}}{\delta}) - c}. \tag{C.8}$$

1) We first consider the limiting case of $\delta \to 0$ for a given $\sigma$. Under the limit, the limiting function $\lim_{\delta \to 0} H(\frac{x^* - \theta}{\delta}) = \begin{cases} 1 & \text{when } \theta < x^* \\ [0,1] & \text{when } \theta = x^* \\ 0 & \text{when } \theta > x^* \end{cases}$. Hence, (5a) can be transformed into

$$\begin{aligned}
& \int_{\theta=\widehat{\theta}}^{\infty} cg(\theta;\mu) d\theta = \int_{\theta=\widehat{\theta}}^{\infty} F \cdot H(\frac{x^* - \theta}{\delta}) g(\theta;\mu) d\theta \\
\Rightarrow \quad & c \left[ 1 - \Phi(\frac{\widehat{\theta} - \mu}{\sigma}) \right] = F \left[ \Phi(\frac{x^* - \mu}{\sigma}) - \Phi(\frac{\widehat{\theta} - \mu}{\sigma}) \right] \\
\Rightarrow \quad & \Phi(\frac{\widehat{\theta} - \mu}{\sigma}) = \frac{F \cdot \Phi(\frac{x^* - \mu}{\sigma}) - c}{F - c},
\end{aligned}$$

which implies

$$\widehat{\theta} < x^*. \tag{C.9}$$

We write $\lim_{\delta \to 0} \widehat{\theta}(x^*; \sigma, \delta) = x^* - m(x^*; \sigma)$, where $m(x^*; \sigma) > 0$ measures the gap between $x^*$ and $\widehat{\theta}$ (independent of $\delta$). With (C.9), (C.7) becomes

$$Y(x^*) = R \cdot \pi(x^*) = \frac{w_0}{\gamma},$$

which clearly has a unique solution with respect to $x^*$. Note that $\lim_{\delta \to 0} Y(x^*; \sigma, \delta) = R \cdot \pi(x^*)$ *pointwise*. A borrowing bank $\theta \in [\widehat{\theta}, x^*)$ faces withdrawals by all its creditors whereas a lending bank $\theta \in [x^*, +\infty)$ has no creditors withdrawing. By (C.8), it follows that

$$I = \frac{(1-c) \cdot X\pi(\widehat{\theta})}{F - c}.$$

In the presence of an interbank market, $\widehat{\theta}$ (or $\widehat{\theta}^i$) and $x^*$ do not coincide when $\delta \to 0$. This is because a bank can survive even if none of its creditors rolls over as long as it can raise no less than $F$ amount of cash (including the funding from the interbank market by collateralizing its cash flow $\mathbb{C}(\theta; x^*)$ at $T_2$). In fact, for a borrowing bank $\theta \in [\widehat{\theta}, x^*)$, it is true that $c + \frac{\mathbb{C}(\theta; x^*)}{I} \geq F$, where $\mathbb{C}(\theta; x^*) = (1-c) \cdot X\pi(\theta)$.

2) We consider the non-limiting case. We prove that when $\delta$ is sufficiently small for a given $\sigma$, (C.7) admits a unique solution with respect to $x^*$ in the interval $x^* \in (-\infty, x^{*U}]$. The proof is similar to the proof of Lemma 1 with three steps.

First, we prove the following result: For any given bounded (finite) interval around $x^* = \mu_\pi$, denoted by $[\mu_\pi - \underline{l}, \mu_\pi + \bar{l}]$, there exists a sufficiently small $\rho > 0$ such that when $\delta < \rho$, function $Y(x^*; \sigma, \delta)$ is monotonically increasing in the interval $x^* \in [\mu_\pi - \underline{l}, \mu_\pi + \bar{l}] \cap D$, where $D$ denotes the domain of function $\widehat{\theta}(x^*; \sigma, \delta)$ with respect to $x^*$.

Based on the result in 1), we write the solution to (5a) in the non-limiting case as

$$\begin{aligned}
\widehat{\theta}(x^*; \sigma, \delta) &= \lim_{\delta \to 0} \widehat{\theta}(x^*; \sigma, \delta) + o(x^*; \sigma, \delta) \\
&= x^* - m(x^*; \sigma) + o(x^*; \sigma, \delta),
\end{aligned}$$

where $\lim_{\delta \to 0} o(x^*; \sigma, \delta) = 0$. Now the gap is $x^* - \widehat{\theta}(x^*; \sigma, \delta) = m(x^*; \sigma) - o(x^*; \sigma, \delta)$. Paralleling (C.2) by changing variables to $z=\frac{\theta-\mu_\theta}{\sigma_\theta}$, function $Y$ in (C.7) is rewritten as

$$\begin{aligned}
Y(x^*;\sigma,\delta) &= \int\limits_{z=\frac{\widehat{\theta}(x^*;\sigma,\delta)-\mu_\theta}{\sigma_\theta}}^{z=+\infty} R\cdot\pi\left(\left(\frac{\alpha}{\alpha+\beta}\mu+\frac{\beta}{\alpha+\beta}x^*\right)+\sqrt{\frac{1}{\alpha+\beta}}z\right)\cdot\phi\left(z\right)dz \\
&= \int\limits_{z=\frac{\frac{\alpha}{\alpha+\beta}(x^*-\mu)-m(x^*;\sigma)+o(x^*;\sigma,\delta)}{\sqrt{\frac{1}{\alpha+\beta}}}}^{z=+\infty} R\cdot\pi\left(\left(\frac{\alpha}{\alpha+\beta}\mu+\frac{\beta}{\alpha+\beta}x^*\right)+\sqrt{\frac{1}{\alpha+\beta}}z\right)\cdot\phi\left(z\right)dz.
\end{aligned}$$

The first-order derivative of $Y(x^*;\sigma,\delta)$ implies

$$\begin{aligned}
\frac{\partial Y(x^*;\sigma,\delta)}{\partial x^*}/R = & \underbrace{-\left[\underline{\pi}+(\overline{\pi}-\underline{\pi})\Phi\left(\frac{x^*-m(x^*;\sigma)+o(x^*;\sigma,\delta)-\mu_\pi}{\sigma_\pi}\right)\right]\cdot\phi\left(\frac{\frac{\alpha}{\alpha+\beta}(x^*-\mu)-[m(x^*;\sigma)-o(x^*;\sigma,\delta)]}{\sqrt{\frac{1}{\alpha+\beta}}}\right)\cdot\frac{\frac{\alpha}{\alpha+\beta}}{\sqrt{\frac{1}{\alpha+\beta}}}}_{<0} \\
& +\underbrace{\int\limits_{z=\frac{\frac{\alpha}{\alpha+\beta}(x^*-\mu)-m(x^*;\sigma)+o(x^*;\sigma,\delta)}{\sqrt{\frac{1}{\alpha+\beta}}}}^{z=+\infty}(\overline{\pi}-\underline{\pi})\phi\left(\frac{\left(\frac{\alpha}{\alpha+\beta}\mu+\frac{\beta}{\alpha+\beta}x^*\right)+\sqrt{\frac{1}{\alpha+\beta}}z-\mu_\pi}{\sigma_\pi}\right)\frac{\frac{\beta}{\alpha+\beta}}{\sigma_\pi}\cdot\phi\left(z\right)dz}_{>0}\,.
\end{aligned} \tag{C.10}$$

We prove that the first term in (C.10) converges *uniformly* to 0 in any given bounded (finite) interval $x^*\in[\mu_\pi-\underline{l},\mu_\pi+\overline{l}]$ when $\delta\to 0$. Because $\frac{\partial\widehat{\theta}}{\partial x^*}>1$ (see the proof of Lemma 3), it is true that $\frac{\partial}{\partial x^*}[x^*-\widehat{\theta}(x^*;\sigma,\delta)]=\frac{\partial}{\partial x^*}[m(x^*;\sigma)-o(x^*;\sigma,\delta)]<0$, implying that the gap $m(x^*;\sigma)-o(x^*;\sigma,\delta)$ is decreasing in $x^*$ and is smallest at $x^*=\mu_\pi+\overline{l}$. Considering $\frac{\alpha}{\alpha+\beta}\to 0$ and $\lim_{\delta\to 0}o(x^*;\sigma,\delta)=0$, then for any arbitrarily small $\varrho>0$, when $\delta$ is sufficiently small, it follows that $\frac{\alpha}{\alpha+\beta}(x^*-\mu)-[m(x^*;\sigma)-o(x^*;\sigma,\delta)]\leq -m(x^*=\mu_\pi+\overline{l};\sigma)+\varrho$ for $x^*\in[\mu_\pi-\underline{l},\mu_\pi+\overline{l}]$. This in turn implies that the term $\phi\left(\frac{\frac{\alpha}{\alpha+\beta}(x^*-\mu)-[m(x^*;\sigma)-o(x^*;\sigma,\delta)]}{\sqrt{\frac{1}{\alpha+\beta}}}\right)$ in the first line of (C.10) converges *uniformly* to 0 in the interval $x^*\in[\mu_\pi-\underline{l},\mu_\pi+\overline{l}]$ when $\delta\to 0$. So does the first term in (C.10) by noting that the additional term $\frac{\frac{\alpha}{\alpha+\beta}}{\sqrt{\frac{1}{\alpha+\beta}}}\to 0$ when $\delta\to 0$.

The second term in (C.10) is bounded from below by a *positive* value for any given finite interval $x^*\in[\mu_\pi-\underline{l},\mu_\pi+\overline{l}]$. Therefore, we can find a sufficiently small $\rho>0$ to make sure that when $\delta<\rho$ the first term of $\frac{\partial Y(x^*;\sigma,\delta)}{\partial x^*}$ in (C.10) is sufficiently close to 0 so that the second term of $\frac{\partial Y(x^*;\sigma,\delta)}{\partial x^*}$ dominates the first term. That is, $\frac{\partial Y(x^*;\sigma,\delta)}{\partial x^*}>0$.

The second and third steps are identical to those in the proof of Lemma 1.

3) The unique equilibrium is also a stable equilibrium. The proof is the same as that of Lemma 1.

**Proof of Proposition 3:** 1) We show that when $\delta$ is high enough (for a given $\sigma$), (C.7) admits multiple solutions with respect to $x^*$ in the interval $x^* \in (-\infty, x^{*U}]$.

First, we prove that for any given $\frac{\delta}{\sigma} > 0$, $\lim\limits_{x^* \to \infty} Y(x^*; \sigma, \delta) \leq 0$. We consider an upper bound function of $Y(x^*; \sigma, \delta)$, denoted by $\overline{Y}(x^*; \sigma, \delta)$:

$$
\begin{aligned}
\overline{Y}(x^*; \sigma, \delta) &\equiv \int_{\theta=\widehat{\theta}(x^*;\sigma,\delta)}^{\theta=+\infty} R \cdot \overline{\pi} \cdot \frac{1}{\sqrt{\frac{1}{\alpha+\beta}}} \phi \left( \frac{\theta - \left( \frac{\alpha}{\alpha+\beta}\mu + \frac{\beta}{\alpha+\beta}x^* \right)}{\sqrt{\frac{1}{\alpha+\beta}}} \right) d\theta \\
&= R \cdot \overline{\pi} \Phi \left( \frac{\left( \frac{\alpha}{\alpha+\beta}\mu + \frac{\beta}{\alpha+\beta}x^* \right) - \widehat{\theta}(x^*; \sigma, \delta)}{\sqrt{\frac{1}{\alpha+\beta}}} \right).
\end{aligned}
$$

Because $\frac{\partial \widehat{\theta}}{\partial x^*} > 1$ and $\frac{\beta}{\alpha+\beta} < 1$, we have $\frac{\alpha}{\alpha+\beta}\mu + \frac{\beta}{\alpha+\beta}x^* - \widehat{\theta}(x^*; \sigma, \delta) \to -\infty$ when $x^* \to \infty$, so $\lim\limits_{x^* \to \infty} \overline{Y}(x^*; \sigma, \delta) = 0$. Hence, $\lim\limits_{x^* \to \infty} Y(x^*; \sigma, \delta) \leq 0$. Note that $\lim\limits_{\delta \to 0} Y(x^*; \sigma, \delta) = R \cdot \pi(x^*)$ pointwise (see the proof of Proposition 2), not globally uniformly.

The derivative $\pi'(\theta)$ achieves its maximum at $\theta = \mu_\pi$ by $\pi'(\theta)|_{\theta=\mu_\pi} = (\overline{\pi} - \underline{\pi})\phi(0)\frac{1}{\sigma_\pi}$. Around $x^* = \mu_\pi$, we have $Y(x^*; \sigma, \delta) > \frac{w_0}{\gamma}$ (under proper parametric values of $c$, $F$ and $R$ to make the research question interesting). Clearly, when $x^*$ is small enough, $Y(x^*; \sigma, \delta) < \frac{w_0}{\gamma}$. Therefore, equation $Y(x^*; \sigma, \delta) = \frac{w_0}{\gamma}$ admits multiple solutions. In particular, when $\frac{\delta}{\sigma}$ is high enough, $Y(x^*; \sigma, \delta) = \frac{w_0}{\gamma}$ admits multiple solutions in interval $x^* \in (-\infty, x^{*U}]$. To see this, $\frac{\beta}{\alpha+\beta}$ is decreasing in $\frac{\delta}{\sigma}$, so when $\frac{\delta}{\sigma}$ is higher, the conditional mean, $\frac{\alpha}{\alpha+\beta}\mu + \frac{\beta}{\alpha+\beta}x^*$, increases more slowly with $x^*$; thus $Y(x^*; \sigma, \delta)$ starts to decline at a lower $x^*$.

In fact, $Y(x^*; \sigma, \delta)$ is typically "$\cap$"- shaped in the relevant region of $x^*$ (i.e., increasing first and then decreasing in $x^*$), in which case equation $Y(x^*; \sigma, \delta) = \frac{w_0}{\gamma}$ can have two solutions; one solution is around $\mu_\pi$, denoted by $x_L^*$, and the other is higher, denoted by $x_H^*$. The numerical simulation confirms the "$\cap$"-shape of function $Y(x^*; \sigma, \delta)$. Figure 5 in the text plots how $Y(x^*; \sigma, \delta)$ evolves when $\delta$ changes.

2) We prove that the lower solution to $Y(x^*) = \frac{w_0}{\gamma}$ corresponds to a stable equilibrium whereas the higher solution corresponds to an unstable equilibrium. Let an individual creditor's threshold

be $x^{*j}$ and that of its peers be $x^*$. So (C.7) can be rewritten as

$$\int_{\theta=\widehat{\theta}(x^*;\sigma,\delta)}^{\theta=+\infty} R\cdot\pi(\theta)\cdot\frac{1}{\sqrt{\frac{1}{\alpha+\beta}}}\phi\left(\frac{\theta-\left(\frac{\alpha}{\alpha+\beta}\mu+\frac{\beta}{\alpha+\beta}x^{*j}\right)}{\sqrt{\frac{1}{\alpha+\beta}}}\right)d\theta=\frac{w_0}{\gamma}.$$

Write the LHS as the function $Y(x^*, x^{*j})$. Based on the result in 1), $Y(x^*)$ is an increasing function at the lower solution $x^* = x^*_L$ and a decreasing function at the higher solution $x^* = x^*_H$. That is, at $(x^*, x^{*j}) = (x^*_L, x^*_L)$, we have $Y(x^* + \triangle, x^{*j} + \triangle) - Y(x^*, x^{*j}) > 0$ for a small positive $\triangle$ and thus $\frac{\partial x^{*j}}{\partial x^*} = -\frac{\partial Y}{\partial x^*}/\frac{\partial Y}{\partial x^{*j}} < 1$. Similarly, at $(x^*, x^{*j}) = (x^*_H, x^*_H)$, we have $Y(x^*+\triangle, x^{*j}+\triangle)-Y(x^*, x^{*j}) < 0$ for a small positive $\triangle$ and thus $\frac{\partial x^{*j}}{\partial x^*} = -\frac{\partial Y}{\partial x^*}/\frac{\partial Y}{\partial x^{*j}} > 1$.

3) The equilibrium of $(x^*, x^{*j}) = (x^*_H, x^*_H)$ is Pareto-dominated by the equilibrium of $(x^*, x^{*j}) = (x^*_L, x^*_L)$ because $\widehat{\theta}_H > \widehat{\theta}_L$. In fact, by $\frac{\partial\widehat{\theta}}{\partial x^*} > 0$ and $\frac{\partial I}{\partial x^*} > 0$ (see the proof of Lemma 3), it follows that $\widehat{\theta}_H > \widehat{\theta}_L$ and $I_H > I_L$.

**Proof of Lemma 2:** Because $x^{*i} = x^*$ in (6) under the symmetric equilibrium, we can replace $x^{*i}$ with $x^*$ in (4a) and (4b).

1) We define function $\Gamma(\theta; x^*, I) \equiv c + \frac{\mathbb{C}(\theta;x^*)}{I} - F\cdot H(\frac{x^*-\theta}{\delta})$ or

$$\Gamma(\theta; x^*, I) = c + \frac{\left[(1-c)X - R\cdot F\cdot\left(1-\Phi(\frac{x^*-\theta}{\delta})\right)\right]\pi(\theta)}{I} - F\cdot\Phi(\frac{x^*-\theta}{\delta}).$$

So equation (4b) is expressed as $\Gamma(\widehat{\theta}^i; x^*, I) = 0$. We are interested in and focus on the case in which the bank survives if and only if its asset quality is above a threshold. We find a sufficient condition to guarantee this — the condition under which the function $\Gamma(\theta; x^*, I)$ is increasing in $\theta$. The first-order derivative of $\Gamma(\theta; x^*, I)$ with respect to $\theta$ is

$$\frac{\partial\Gamma(\theta; x^*, I)}{\partial\theta} = \frac{\left[(1-c)X - R\cdot F\cdot\left(1-\Phi(\frac{x^*-\theta}{\delta})\right)\right]\pi'(\theta)}{I} + \frac{F}{\delta}\cdot\phi(\frac{x^*-\theta}{\delta})\left[1-\frac{R\cdot\pi(\theta)}{I}\right].$$

Thus, a sufficient condition for $\frac{\partial\Gamma(\theta;x^*,I)}{\partial\theta} > 0$ is $I > R\overline{\pi}$ by noting that $\pi(\theta) \le \overline{\pi}$ for any $\theta$ and thus the second term is positive whereas the first term is also positive by (A.2). Under the sufficient condition of $I > R\overline{\pi}$, we also have

$$\frac{\partial\Gamma(\theta; x^*, I)}{\partial x^*} = -\frac{F}{\delta}\cdot\phi(\frac{x^*-\theta}{\delta})\left[1-\frac{R\cdot\pi(\theta)}{I}\right] < 0.$$

In addition, it is straightforward to show $\frac{\partial\Gamma(\theta;x^*,I)}{\partial I} < 0$.

Note that when creditors play the creditor-run game, they form an expectation on $I$ and takes it as given and they also know $R$ (which is set at $T_0$). We focus on the case where in the full equilibrium of the model it is true that $I > R\overline{\pi}$. Non-restrictive parameter values (e.g., $\overline{\pi}$ is small enough and $X$ is big enough by recalling equilibrium condition $FR < (1-c)X$ in (A.2)) can guarantee that $I > R\overline{\pi}$ in the full equilibrium. The numerical example in Appendix B illustrates one case under a set of parameter values.

2) We prove that the equilibrium is unique when $\frac{\delta}{\sigma}$ is small enough. Equation (4b) gives a unique $\widehat{\theta}^i$ for a given $x^*$ and $I$; denote the solution by $\widehat{\theta}^i(x^*; I)$. By applying the implicit function theorem to equation $\Gamma(\widehat{\theta}^i; x^*, I) = 0$, we have $\frac{\partial \widehat{\theta}^i}{\partial I} = -\frac{\partial \Gamma / \partial I}{\partial \Gamma / \partial \widehat{\theta}^i} > 0$ and

$$\frac{\partial \widehat{\theta}^i}{\partial x^*} = -\frac{\partial \Gamma / \partial x^*}{\partial \Gamma / \partial \widehat{\theta}^i} \in (0, 1] .$$

This result is stronger than its counterpart in Lemma 1 where $\frac{\partial \widehat{\theta}}{\partial x^*} = 1$, so the unique threshold equilibrium is more likely here than in Lemma 1. Concretely, (4a) becomes

$$\int_{\theta=\widehat{\theta}^i(x^*;I)}^{\theta=+\infty} R \cdot \pi(\theta) \cdot \frac{1}{\sqrt{\frac{1}{\alpha+\beta}}} \phi \left( \frac{\theta - \left( \frac{\alpha}{\alpha+\beta}\mu + \frac{\beta}{\alpha+\beta}x^* \right)}{\sqrt{\frac{1}{\alpha+\beta}}} \right) d\theta = \frac{w_0}{\gamma}. \tag{C.11}$$

Write the LHS of (C.11) as function $Y(x^*; I)$. Paralleling (C.2) by changing variables to $z = \frac{\theta - \mu_\theta}{\sigma_\theta}$, function $Y$ in (C.11) is rewritten as

$$Y(x^*; I) = \int_{z=\frac{\widehat{\theta}^i(x^*;I) - \left( \frac{\alpha}{\alpha+\beta}\mu + \frac{\beta}{\alpha+\beta}x^* \right)}{\sqrt{\frac{1}{\alpha+\beta}}}}^{z=+\infty} R \cdot \pi \left( \left( \frac{\alpha}{\alpha+\beta}\mu + \frac{\beta}{\alpha+\beta}x^* \right) + \sqrt{\frac{1}{\alpha+\beta}} z \right) \cdot \phi(z)\, dz \ .$$

The first-order derivative of $Y(x^*; I)$ implies

$$
\frac{\partial Y(x^*;\sigma,\delta)}{\partial x^*}/R = \underbrace{\begin{array}{c} -\pi\left( (\frac{\alpha}{\alpha+\beta}\mu + \frac{\beta}{\alpha+\beta}x^*) + \sqrt{\frac{1}{\alpha+\beta}}\frac{\widehat{\theta}^i(x^*;I)-(\frac{\alpha}{\alpha+\beta}\mu+\frac{\beta}{\alpha+\beta}x^*)}{\sqrt{\frac{1}{\alpha+\beta}}}\right) \\ \cdot\phi\left(\frac{\widehat{\theta}^i(x^*;I)-(\frac{\alpha}{\alpha+\beta}\mu+\frac{\beta}{\alpha+\beta}x^*)}{\sqrt{\frac{1}{\alpha+\beta}}}\right)\cdot \frac{\frac{\partial\widehat{\theta}^i}{\partial x^*}-\frac{\beta}{\alpha+\beta}}{\sqrt{\frac{1}{\alpha+\beta}}} \end{array}}_{+/-}
$$

$$
+\underbrace{\int\limits_{z=\frac{\widehat{\theta}^i(x^*;I)-\left(\frac{\alpha}{\alpha+\beta}\mu+\frac{\beta}{\alpha+\beta}x^*\right)}{\sqrt{\frac{1}{\alpha+\beta}}}}^{z=+\infty} (\overline{\pi}-\underline{\pi})\phi\left(\frac{\left(\frac{\alpha}{\alpha+\beta}\mu+\frac{\beta}{\alpha+\beta}x^*\right)+\sqrt{\frac{1}{\alpha+\beta}}z-\mu_\pi}{\sigma_\pi}\right)\frac{\frac{\beta}{\alpha+\beta}}{\sigma_\pi}\cdot\phi(z)\,dz}_{>0}\ . \tag{C.12}
$$

In the first term of (C.12), $\frac{\partial\widehat{\theta}^i}{\partial x^*}\leq 1$ while $\frac{\beta}{\alpha+\beta}\to 1$ when $\delta\to 0$. So, either $\frac{\partial\widehat{\theta}^i}{\partial x^*}=1$ in which case the term $\frac{\frac{\partial\widehat{\theta}^i}{\partial x^*}-\frac{\beta}{\alpha+\beta}}{\sqrt{\frac{1}{\alpha+\beta}}}\to 0$ when $\delta\to 0$, or $\frac{\partial\widehat{\theta}^i}{\partial x^*}<1$ in which case $\frac{\frac{\partial\widehat{\theta}^i}{\partial x^*}-\frac{\beta}{\alpha+\beta}}{\sqrt{\frac{1}{\alpha+\beta}}}<0$ when $\delta\to 0$. Therefore, similar to the proof of Lemma 1, there exists a sufficiently small $\rho>0$ such that when $\delta<\rho$, function $Y(x^*;I)$ is monotonically increasing in any given finite interval $x^*\in[\mu_\pi-\underline{l},\mu_\pi+\overline{l}]$. By using the second and third steps in the proof of Lemma 1, we have the overall result that when $\frac{\delta}{\sigma}$ is small enough, equation $Y(x^*;I)=0$ admits a unique solution with respect to $x^*$ in the interval $x^*\in(-\infty,x^{*U}]$.

3) We prove the comparative statics $\frac{\partial x^*}{\partial I}>0$ for a stable equilibrium. As shown in the proofs of Lemma 1 and Proposition 3, a stable equilibrium corresponds to $Y(x^*;I)$ being increasing in $x^*$; that is, $\frac{\partial Y(x^*;I)}{\partial x^*}>0$. Since $\frac{\partial\widehat{\theta}^i}{\partial I}>0$, it is easy to prove that $\frac{\partial Y(x^*;I)}{\partial I}<0$. Therefore, $\frac{\partial x^*}{\partial I}=-\frac{\partial Y(x^*;I)/\partial I}{\partial Y(x^*;I)/\partial x^*}>0$.

**Proof of Corollary 1:** From Lemma 2, (4a)-(4b) give $x^{*i}$ for each expectation of $I$. By (6), $x^{*i}=x^*$. So (4a)-(4b) and (6) together give $x^*$ for each expectation of $I$. By Lemma 3, (5a)-(5b) determine a unique $I$ for a given $x^*$ . Therefore, the creditor run-interbank market equilibrium at $T_1$ is characterized by the fixed point problem between $x^*(I)$ and $I(x^*;\mu)$.

**Proof of Corollary 2:** Rewrite (C.7) as

$$
\int\limits_{\theta=\widehat{\theta}(x^*;\mu,\sigma,\delta,c)}^{\theta=+\infty} R\cdot\pi(\theta)\cdot\frac{1}{\sqrt{\frac{1}{\alpha+\beta}}}\phi\left(\frac{\theta-\left(\frac{\alpha}{\alpha+\beta}\mu+\frac{\beta}{\alpha+\beta}x^*\right)}{\sqrt{\frac{1}{\alpha+\beta}}}\right)d\theta=\frac{w_0}{\gamma}.
$$

Write LHS as function $Y(x^*;\mu,\sigma,\delta,c)$. Considering that $\frac{\partial\widehat{\theta}}{\partial\mu}<0$ and $\frac{\partial\widehat{\theta}}{\partial c}<0$ (see the proof of Lemma 3), we have $\frac{\partial Y(x^*;\mu,\sigma,\delta,c)}{\partial\mu}>0$ and $\frac{\partial Y(x^*;\mu,\sigma,\delta,c)}{\partial c}>0$. Hence, when $\mu$ or $c$ decreases, the curve of $Y(x^*;\mu,\sigma,\delta,c)$ shifts downward. Thus, the larger solution to equation $Y(x^*;\mu,\sigma,\delta,c)=\frac{w_0}{\gamma}$ becomes smaller and consequently is more likely to fall below $x^{*U}$. That is, the existence of a second (unstable) equilibrium becomes more likely.

Figure C-1 plots $Y(x^*;\mu,\sigma,\delta,c)$ when $\mu$ changes (where parameter values except $\mu$ are the ones in the numerical example in Appendix B).

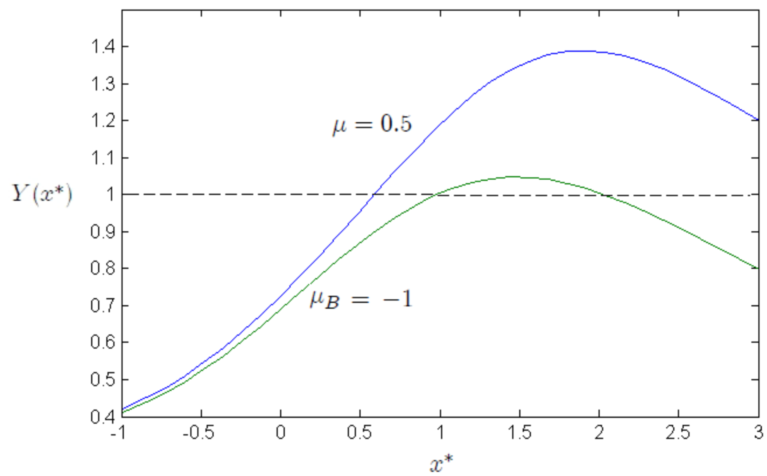


**Figure C-1**: Function $Y(x^*;\mu,\sigma,\delta,c)$ when $\mu$ changes

**Proof of Proposition 4:** Based on the proof of Proposition 2 and Corollary 2, when the shock, $\mu-\mu_\varepsilon$, is small, the new equilibrium is also unique. Write the solution in (4a)-(4b) and (6) as the reaction function $x^*(I;\mu)$, where $\mu$ is public information. For a stable equilibrium given by (4a)-(4b), we have shown that $\frac{\partial x^*}{\partial I}>0$ and it is easy to show that $\frac{\partial x^*}{\partial\mu}<0$. We write the solution in (5a)-(5b) as the reaction function $I(x^*;u)$ where $\mu$ is the mean of the asset quality distribution. We have shown that $\frac{\partial I}{\partial x^*}>0$ and $\frac{\partial I}{\partial\mu}<0$. Therefore, a negative shock of $\mu$ leads to $x^*$ and $I$ spiraling upward. Certainly, $\widehat{\theta}_{NB}>\widehat{\theta}_N$, $x^*_{NB}>x^*_N$ and $I_{NB}>I_N$.

**Proof of Proposition 5:** Based on Propositions 2 and 3 and Corollary 2, the proof is straightforward.

**Proof in the subsection on liquidity injections in Section 6:** We prove that the pure promise of bailout in our model cannot achieve the same effect of deposit insurance in Diamond and Dybvig (1983) as a costless solution to liquidity crises.

In our model, the banks have insolvency issues at $T_2$ (i.e., the long-term asset of a bank pays 0 or $X$). So it is impossible for the government to insure creditors' payoff $R$ at $T_2$ always without an actual bailout. Therefore, the government can at most insure creditors' payoff, 1, at $T_1$. Now we show that such an insurance necessarily requires some actual liquidity injections of the government.

Suppose creditors of a bank believe that their bank will always survive to $T_2$ and has no interim illiquidity risk (coordination risk) at all at $T_1$ (i.e., $\widehat{\theta} = -\infty$). Then they use the lowest rollover threshold $x^* = \underline{x}$, where $\underline{x}$ solves

$$\int_{-\infty}^{\infty} (\gamma R) \cdot \pi(\theta) \cdot h_g(\theta | x^* = \underline{x}) d\theta = w_0.$$

Given $x^* = \underline{x}$, a bank of quality $\theta$ needs liquidity in amount of $F \cdot H(\frac{\underline{x}-\theta}{\delta})$ at $T_1$ to satisfy its creditors' withdrawals. If the bank does not *actually* need the government's liquidity support and has enough liquidity of its own, it requires

$$c \geq F \cdot H(\frac{\underline{x} - \theta}{\delta}),$$

which is not true for a sufficiently small $\theta$. In other words, even if creditors feel very secure and set $x^* = \underline{x}$ (the lowest threshold possible), it is impossible for all banks to satisfy their creditors' early withdrawals on their own. Some banks must *actually* rely on the government's liquidity support. In fact, only if the government promises and commits to supporting a subset of banks of high quality such as $\theta \in [\theta^U, \infty]$ will the bailout not actually be needed in equilibrium, which is the case of the upper dominance region.

**Proof of Proposition 6:** With public disclosure, the equilibrium at $T_1$ under the shock (i.e., state $s = B$) is given by the system of equations (4a)-(4b), (5a)-(5b) and (6), where the allocation $(c, R)$ is $(c_N, R_N)$, the asset quality distribution $g(\cdot)$ in (5a) is replaced by $g_B(\cdot)$, and the posterior conditional density $h_g(\theta | x^{*i})$ in (4a) is replaced by (7).

Hence, based on (C.7) in the proof of Proposition 2, the equilibrium at $T_1$ under public disclosure is given by

$$\int_{\theta=\widehat{\theta}(x^*;\sigma,\delta)}^{\theta=\infty} R \cdot \pi(\theta) \cdot \hat{h}_g(\theta | x^*, \lambda) d\theta = \frac{w_0}{\gamma} \tag{C.13}$$

where

$$\hat{h}_g(\theta|x^*,\lambda) = \frac{\frac{1}{\sigma}\phi(\frac{\theta-\mu_B}{\sigma})\frac{1}{\delta}\phi(\frac{x^*-\theta}{\delta})}{\int\limits_{\theta=\underline{\theta}(\lambda)}^{\theta=+\infty}\frac{1}{\sigma}\phi(\frac{\theta-\mu_B}{\sigma})\frac{1}{\delta}\phi(\frac{x^*-\theta}{\delta})d\theta} = \frac{\frac{1}{\sqrt{\frac{1}{\alpha+\beta}}}\phi\left(\frac{\theta-\left(\frac{\alpha}{\alpha+\beta}\mu_B+\frac{\beta}{\alpha+\beta}x^*\right)}{\sqrt{\frac{1}{\alpha+\beta}}}\right)}{\Phi\left(\frac{\left(\frac{\alpha}{\alpha+\beta}\mu_B+\frac{\beta}{\alpha+\beta}x^*\right)-\underline{\theta}(\lambda)}{\sqrt{\frac{1}{\alpha+\beta}}}\right)} \text{ for } \theta \geq \underline{\theta}(\lambda)$$

and $\underline{\theta}(\lambda)$ is the solution to $\Phi(\frac{\theta-\mu_B}{\sigma}) = 1-\lambda$. Clearly, when $\lambda = 1$ or $\underline{\theta} = -\infty$, $\hat{h}_g(\theta|x^*,\lambda)$ becomes $h_g(\theta|x^*)$.

Write the LHS of (C.13) as function $Y(x^*;\sigma,\delta,\lambda)$. We prove that $\frac{\partial Y}{\partial x^*} > 0$ in the lower equilibrium and $\frac{\partial Y}{\partial \lambda} < 0$. It is straightforward to see $\frac{\partial Y}{\partial \lambda} < 0$. Based on the proof of Proposition 2, to prove $\frac{\partial Y}{\partial x^*} > 0$ is to prove that distribution $\hat{h}_g(\theta|x_1^*,\lambda)$ has first-order stochastic dominance over distribution $\hat{h}_g(\theta|x_2^*,\lambda)$ for $x_1^* > x_2^*$. Denote the c.d.f. of $\hat{h}_g(\theta|x^*,\lambda)$ by $\hat{H}_g(\theta|x^*,\lambda)$ and thus

$$\hat{H}_g(\theta|x^*,\lambda) = \frac{\Phi\left(\frac{\theta-\left(\frac{\alpha}{\alpha+\beta}\mu_B+\frac{\beta}{\alpha+\beta}x^*\right)}{\sqrt{\frac{1}{\alpha+\beta}}}\right) - \Phi\left(\frac{\underline{\theta}(\lambda)-\left(\frac{\alpha}{\alpha+\beta}\mu_B+\frac{\beta}{\alpha+\beta}x^*\right)}{\sqrt{\frac{1}{\alpha+\beta}}}\right)}{1-\Phi\left(\frac{\underline{\theta}(\lambda)-\left(\frac{\alpha}{\alpha+\beta}\mu_B+\frac{\beta}{\alpha+\beta}x^*\right)}{\sqrt{\frac{1}{\alpha+\beta}}}\right)} \text{ for } \theta \geq \underline{\theta}(\lambda).$$

Hence, we need to prove $\frac{\partial \hat{H}_g(\theta|x^*,\lambda)}{\partial x^*} < 0$ for every $\theta$. This is true because

$$\begin{aligned} sgn\left(\frac{\partial \hat{H}_g(\theta|x^*,\lambda)}{\partial x^*}\right) &= sgn\left[\frac{\phi\left(\frac{\underline{\theta}(\lambda)-\left(\frac{\alpha}{\alpha+\beta}\mu_B+\frac{\beta}{\alpha+\beta}x^*\right)}{\sqrt{\frac{1}{\alpha+\beta}}}\right)}{1-\Phi\left(\frac{\underline{\theta}(\lambda)-\left(\frac{\alpha}{\alpha+\beta}\mu_B+\frac{\beta}{\alpha+\beta}x^*\right)}{\sqrt{\frac{1}{\alpha+\beta}}}\right)} - \frac{\phi\left(\frac{\theta-\left(\frac{\alpha}{\alpha+\beta}\mu_B+\frac{\beta}{\alpha+\beta}x^*\right)}{\sqrt{\frac{1}{\alpha+\beta}}}\right)}{1-\Phi\left(\frac{\theta-\left(\frac{\alpha}{\alpha+\beta}\mu_B+\frac{\beta}{\alpha+\beta}x^*\right)}{\sqrt{\frac{1}{\alpha+\beta}}}\right)}\right] \\ &= -1, \end{aligned}$$

where the second line follows by the monotone hazard rate (MHR) of the normal distribution.

Therefore, by the implicit function theorem, $\frac{\partial x^*}{\partial \lambda} = -\frac{\partial Y}{\partial \lambda}/\frac{\partial Y}{\partial x^*} > 0$. Based on the proof of Lemma 3, $\frac{\partial \widehat{\theta}}{\partial x^*} > 0$. Hence $\frac{\partial \widehat{\theta}}{\partial \lambda} > 0$. When $\lambda$ starts from 1 and decreases, $\widehat{\theta}$ decreases while $\underline{\theta}$ increases. Considering the constraint $\underline{\theta} \leq \widehat{\theta}$, there is a unique $\lambda$ such that $\underline{\theta}$ and $\widehat{\theta}$ coincide, at which $\widehat{\theta}$ is minimized.

**Proof of Lemma 4:** 1) We work out the *ex post* payoff to the equityholder (of the equilibrium outcome) in the creditor-run game. The ex post payoff to the equityholder in a bank of quality $\theta$ at $T_1$ is calculated as follows. If the bank fails (i.e., $\theta < \widehat{\theta}$) at $T_1$, its equityholder gets nothing. If the bank survives (i.e., $\theta \geq \widehat{\theta}$) at $T_1$, the equityholder's payoff includes his claim on the long-term asset as well as his position in the interbank market as follows:

$$\underbrace{\left[c - F \cdot H(\frac{x^* - \theta}{\delta})\right] I}_{\text{Gain or loss in interbank market}} + \underbrace{\left[(1-c)X - R \cdot F \cdot \left(1 - H(\frac{x^* - \theta}{\delta})\right)\right] \pi(\theta)}_{\text{Payoff in risky investment}}. \tag{C.14}$$

(C.14) is easy to understand based on Figure 4. The first term, which can be positive (i.e., for lending) or negative (i.e., for borrowing), is the bank's gain or loss in the interbank market, and the second term is the expected payoff of its long-term asset net of the claim by its staying creditors. In fact, for the marginal bank with $\theta = \widehat{\theta}$, the equity value in (C.14) is exactly equal to 0. A bank of quality $\theta > \widehat{\theta}$ has a positive expected equity value.

We illustrate that the aggregate value in the objective function of Program A1 is equal to the sum of the equityholder's payoff across banks in (C.14) plus the sum of debtholders' payoffs across banks on the RHS of (A.1'). By (C.14), the aggregate equity value is

$$\begin{aligned} & \int_{\widehat{\theta}}^{\infty} \left\{ \left[c - F \cdot H(\frac{x^* - \theta}{\delta})\right] I + \left[(1-c)X - R \cdot F \cdot \left(1 - H(\frac{x^* - \theta}{\delta})\right)\right] \pi(\theta) \right\} g(\theta) d\theta \\ = & \int_{\widehat{\theta}}^{\infty} \left[(1-c)X - R \cdot F \cdot \left(1 - H(\frac{x^* - \theta}{\delta})\right)\right] \pi(\theta) \cdot g(\theta) d\theta \end{aligned} \tag{C.15}$$

The second line in the above is obtained because the first term in the first line is cancelled out by (5a), i.e., $\int_{\widehat{\theta}}^{\infty} c g(\theta) d\theta = \int_{\widehat{\theta}}^{\infty} F \cdot H(\frac{x^* - \theta}{\delta}) g(\theta) d\theta$. Intuitively, the gains and losses in the interbank market across banks cancel each other out. By (A.1'), the aggregate debt value is

$$\int_{-\infty}^{\widehat{\theta}} c \cdot g(\theta) d\theta + \int_{\widehat{\theta}}^{\infty} \left[F \cdot H(\frac{x^* - \theta}{\delta}) + R \cdot F \cdot \left(1 - H(\frac{x^* - \theta}{\delta})\right) \cdot \pi(\theta)\right] g(\theta) d\theta \tag{C.16}$$

Therefore, the aggregate value in the economy is the sum of the terms of (C.15) and (C.16), which by using (5a) again becomes

$$\int_{-\infty}^{\widehat{\theta}} c \cdot g(\theta) d\theta + \int_{\widehat{\theta}}^{\infty} \left[c + (1-c)\left(X \cdot \pi(\theta)\right)\right] g(\theta) d\theta.$$

2) We turn to Lemma 4. First, for a given $c$, the system of equations (4a)-(4b), (5a)-(5b), (6) and (A.1') determines the vector $(R, \widehat{\theta}, x^*, I)$. In fact, based on Propositions 2 and 3, for a given $c$ and $R$, the pair $(x^*, \widehat{\theta})$ is determined. Plugging functions $x^*(c, R)$ and $\widehat{\theta}(c, R)$ into (A.1'), we can solve equation (A.1') with respect to $R$. Overall, we can find $\widehat{\theta}$ as a function of $c$.

Second, by plugging the function of $\widehat{\theta}(c)$ into the bank value (denoted by $V$) in the objective function in Program A1, we obtain $V$ as a function of $c$. Because all functions are continuous, the optimization of $V$ on the close set $[0, 1]$ clearly has solutions.

Third, for typical cases (to make the research questions interesting), the optimal $c$ cannot be such that $\widehat{\theta} = -\infty$. Denote by $\bar{c}$ the cash holdings to achieve $\widehat{\theta} = -\infty$. We prove that $\frac{\partial V}{\partial c}|_{c=\bar{c}} < 0$. In fact,

$$\begin{aligned}\frac{\partial V}{\partial c}|_{c=\bar{c}} &= \left[(1-c)(X \cdot \pi(\widehat{\theta}))\frac{dG(\widehat{\theta})}{dc}\right]|_{c=\bar{c},\ \widehat{\theta}=-\infty} - \left[\int_{-\infty}^{\infty}(X \cdot \pi(\theta))g(\theta)d\theta - 1\right] \\ &= \left[(1-c)(\underline{\pi} \cdot X)\frac{dG(\widehat{\theta})}{dc}\right]|_{c=\bar{c},\ \widehat{\theta}=-\infty} - \left(\frac{\underline{\pi}+\overline{\pi}}{2}X - 1\right).\end{aligned}$$

When $\underline{\pi} \to 0$, the first term in the second line above approaches 0, in which case $\frac{\partial V}{\partial c}|_{c=\bar{c}} < 0$ for $\frac{\underline{\pi}+\overline{\pi}}{2}X - 1 > 0$. Note that we assume $\int_{-\infty}^{\infty}[X \cdot \pi(\theta)]\, g(\theta)d\theta > R_0$ where $R_0 \geq 1$, which means that $\frac{\underline{\pi}+\overline{\pi}}{2}X > 1$. Therefore, under the sufficient condition that $\underline{\pi}$ is small enough, $\frac{\partial V}{\partial c}|_{c=\bar{c}} < 0$.

**Proof of Lemma 5:** The proof is similar to the Proof of Lemma 4. First, for a given $c$, the system of equations (A.3) and (A.4) determines the vector $(R, \widehat{\theta}_s, x_s^*, I_s)$ for $s = N$ and $B$. In fact, based on Propositions 2 and 3, for a given $c$ and $R$, the pair $(x_s^*, \widehat{\theta}_s)$ is determined under a realized state $s$. Plugging functions $x_s^*(c, R)$ and $\widehat{\theta}_s(c, R)$ for $s = N$ and $B$ into (A.4), we can solve equation (A.4) with respect to $R$. Overall, we can find $\widehat{\theta}_s$ as a function of $c$.

Second, by plugging the function of $\widehat{\theta}_s(c)$ into the bank value (denoted by $V$) in the objective function in Program A2, we obtain $V$ as a function of $c$. Because all functions are continuous, the optimization of $V$ on the close set $[0, 1]$ clearly has solutions. Also, because all functions are continuous and bounded, we have that when $q \to 1$, $(c, R) \to (c_N, R_N)$ and that when $q \to 0$, $(c, R) \to (c_B, R_B)$.

# D More explanations

## D. 1 Alternative assumption on debt seniority

We show that under the alternative assumption that interbank lenders are senior to depositors (creditors), the equilibrium at $T_1$ does not change qualitatively. First, for the creditor-run equilibrium of an individual bank, (4a)-(4b) is replaced by (D.1)-(D.2)

$$\int_{-\infty}^{\widehat{\theta}^i} 0 \cdot h_g(\theta|x^{*i})d\theta + \int_{\widehat{\theta}^i}^{\infty} \min\left[\frac{(1-c)X - \frac{\left[F \cdot H(\frac{x^{*i}-\theta}{\delta})-c\right]\ I}{\pi(\theta)}}{F \cdot \left(1 - H(\frac{x^{*i}-\theta}{\delta})\right)}, R\right] \pi(\theta)h_g(\theta|x^{*i})d\theta = \frac{w_0}{\gamma} \tag{D.1}$$

$$\frac{[(1-c)X]\,\pi(\widehat{\theta}^i)}{I} + c = F \cdot H(\frac{x^{*i} - \widehat{\theta}^i}{\delta}). \tag{D.2}$$

The comparative statics result $\frac{\partial x^{*i}}{\partial I} \geq 0$ still holds. In fact, a higher $I$ not only makes the bank less liquid at $T_1$ (i.e., a higher $\widehat{\theta}^i$) but also results in less value left for the staying creditors because the interbank repayment is senior; the two forces lead to a higher $x^{*i}$. Second, for the interbank market equilibrium, (5a)-(5b) do not change.

## D. 2 Comparison between autarky and the presence of an interbank market

The presence of an interbank market (versus autarky) affects not only the equilibrium at $T_1$ but also the allocation $(c, R)$ at $T_0$ in the first place. In what follows, we show: 1) the presence of an interbank market leads to ex-post better liquidity sharing. So if an efficient equilibrium ex post at $T_1$ is always selected (e.g., with ex-post regulation), the existence of an interbank market dominates autarky in improving the overall welfare; 2) however, because of the ex-post better liquidity sharing, the ex-ante allocation $(c, R)$ becomes more “aggressive” in the sense that $c$ is lower in the presence of an interbank market than in autarky. This, together with the higher degree of strategic complementarity among creditors in the presence of an interbank market, can make equilibrium multiplicity more likely at $T_1$ and thus increase the fragility. The above effect is illustrated in Figure D-1, where $(x_L^*, x_L^*)|_M$ and $(x_H^*, x_H^*)|_M$ denote the equilibria at $T_1$ in the presence of an interbank market and $(x_L^*, x_L^*)|_A$ denotes the equilibrium in autarky.

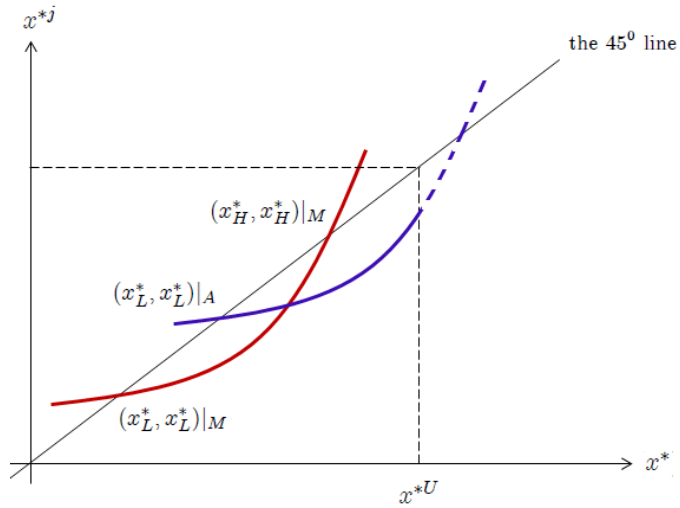


**Figure D-1**: Equilibria at $T_1$ in the presence of an interbank market and in autarky

First, we show that if an efficient equilibrium ex post is selected, the presence of an interbank market achieves a higher social welfare than autarky. The economic mechanism is that the interbank market leads to both ex post better liquidity sharing and hence ex ante more efficient portfolio allocations. Concretely, in autarky, the optimal amount of cash holdings for banks is given by

$$\max_c \int_{-\infty}^{\widehat{\theta}} c \cdot g(\theta) d\theta + \int_{\widehat{\theta}}^{\infty} [c + (1-c)(X \cdot \pi(\theta))]\, g(\theta) d\theta \qquad \text{(Program D1)}$$
$$\text{s.t. (1)-(2), (A.1') and (A.2)}$$

In autarky, banks are i.i.d. and thus the marginal bank threshold $\widehat{\theta}$ in the system is equal to the failure threshold $\widehat{\theta}$. We need to show that the maximum bank value given by Program A1 is higher than that given by Program D1. First, for a same $c$, Program A1 achieves a higher value than Program D1. In fact, in comparing (2) with (5a), (2) means that every surviving bank *at least* holds cash of an amount $c$, whereas (5a) means that surviving banks *on average* hold an amount $c$. Hence, for a given $c$, including the one that optimizes Program D1, $\widehat{\theta}(c)$ is lower in Program A1 than in Program D1. Second, $c$ in Program A1 can be different from and actually is lower than that in Program D1 to achieve an even higher maximum bank value.

Second, with a lower $c$, the presence of an interbank market can make equilibrium multiplicity more likely at $T_1$. The key reason is that the degree of strategic complementarities among creditors is stronger with an interbank market than without one. Let an individual creditor's threshold be

$x^{*j}$ and that of its peers be $x^*$. So equations (C.1) and (C.7) can be rewritten in one form:

$$\int_{\theta=\widehat{\theta}(x^*)}^{\theta=+\infty} R\cdot\pi(\theta)\cdot\frac{1}{\sqrt{\frac{1}{\alpha+\beta}}}\phi\left(\frac{\theta-\left(\frac{\alpha}{\alpha+\beta}\mu+\frac{\beta}{\alpha+\beta}x^{*j}\right)}{\sqrt{\frac{1}{\alpha+\beta}}}\right)d\theta=\frac{w_0}{\gamma}, \tag{D.3}$$

where $\widehat{\theta}(x^*)$ is defined by (2) for (C.1) and by (4a) for (C.7). We write the LHS of (D.3) as function $Y(x^*, x^{*j})$. Hence,

$$\frac{\partial x^{*j}}{\partial x^*}=-\frac{\partial Y}{\partial x^*}\Big/\frac{\partial Y}{\partial x^{*j}}=\frac{d\widehat{\theta}}{dx^*}\left(\frac{-\frac{\partial Y}{\partial\widehat{\theta}}}{\frac{\partial Y}{\partial x^{*j}}}\right).$$

We set the following proper benchmark for comparison: let the different levels of $c$ in the presence of an interbank market and in autarky be such that they achieve the same level of $\widehat{\theta}$, namely, $\widehat{\theta}|_{\text{interbank}}=\widehat{\theta}|_{\text{autarky}}$. So $\frac{-\frac{\partial Y}{\partial\widehat{\theta}}}{\frac{\partial Y}{\partial x^{*j}}}|_{\text{interbank}}=\frac{-\frac{\partial Y}{\partial\widehat{\theta}}}{\frac{\partial Y}{\partial x^{*j}}}|_{\text{autarky}}$. The proof of Lemma 3 has shown that the following is true:

$$\frac{d\widehat{\theta}}{dx^*}|_{\text{interbank}}>1.$$

Because $\frac{d\widehat{\theta}}{dx^*}|_{\text{autarky}}=1$ by (2), we have

$$\frac{d\widehat{\theta}}{dx^*}|_{\text{interbank}}>\frac{d\widehat{\theta}}{dx^*}|_{\text{autarky}}\Longrightarrow\frac{\partial x^{*j}}{\partial x^*}|_{\text{interbank}}>\frac{\partial x^{*j}}{\partial x^*}|_{\text{autarky}}. \tag{D.4}$$

(D.4) means that the degree of strategic complementarity among creditors is stronger with an interbank market than without one. This in turn implies that the existence of a second (unstable) equilibrium (at which $\frac{\partial x^{*j}}{\partial x^*}>1$) is more likely with the interbank market than in autarky.

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